Chapter 2: PROBABILITY

Part 4: Conditional Probability, Multiplication and Total Probability Rules

Sections 2-4, 2-5

Sometimes probabilities need to be re-evaluated or adjusted as additional information becomes available that is specific to the situation.

- **Conditional Probability**

  The probability of an event B under the knowledge that the outcome will be in event A is denoted as $P(B|A)$ and

$$P(B|A) \;=\; \frac{P(A \cap B)}{P(A)} \qquad \text{for} \qquad P(A) > 0$$

  Venn diagram:

Example: Sex and Rank in STEM faculty at ISU

- Randomly choose a STEM faculty member.

  Let A denote the event that the faculty member is an assistant professor.
  Let B denote the event that the faculty member is is female.

Frequency table of 518 STEM faculty members:

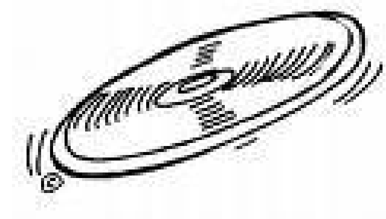
RANK

SEX		Assistant	Associate	Full
	Male	98	87	254
	Female	36	25	18

P(female | they are an assistant professor)=

$$P(B|A) \quad = \quad \frac{P(A \cap B)}{P(A)} = \frac{36/518}{134/518} = 36/134 = 0.2687$$

Mosaic Plot:



Example: Manufactured Parts
Company A

- Sometimes a superficial flaw on a part may signal that there's a fatal error with a product, but sometimes a part with a superficial flaw may function just fine.

Can the error that we see on the surface help us know which parts are actually defective?

In a manufacturing process,
10% of the parts contain visible flaws
90% of the parts do not contain visible flaws

Of the parts containing a visible flaw, 25% are defective
Of the parts not containing a visible flaw, 5% are defective

The chance of being defective depends on 'visible flaw' status.

Mosaic plot:

Define the events and probabilities:

Let D be the event that a part is defective
Let F be the event that a part has a superficial flaw

$P(F) = 0.10$
$P(F') = 0.90$

$P(D|F) = 0.25$
$P(D|F') = 0.05$

Example: Manufactured Parts
Company B

- A biomedical engineering company has 2 lines which produce it's prosthetic legs. Information on the 553 legs produced from two weeks is given in the following frequency table:

	line 1	**line 2**	**total**
defective	6	1	7
not defective	345	201	546
total	351	202	553

Let D be the event that a part is defective
Let L be the event that a part came from line 1

$P(L) =$ $\qquad$ $P(L') =$

$P(D|L) =$ $\qquad$ $P(D|L') =$

e.g. (cont.)

We can also consider these conditional probabilities by the definition of conditional probability.

$$P(D|L) \quad = \quad \frac{P(D \cap L)}{P(L)} = \underline{\hspace{4cm}}$$

Note that

$$P(D) \qquad \text{and} \qquad P(D|L)$$

are both probabilities of getting a defective leg, but they are calculated under different situations, or states of knowledge.

Random sampling and conditional probabilities

- **Random Samples**

  To select randomly implies that at each step of the sample, the items that remain in the batch are equally likely to be selected.

Example: Sequential random draws without replacement

Suppose you are to choose two items from a bin at random. In the bin are 35 Grade 2 hex bolts, and 15 Grade 5 hex bolts.

1) What is the probability of choosing a Grade 5 hex bolt on the second draw if you drew a Grade 2 hex bolt on the first draw?

2) What is the probability of choosing a Grade 5 first, and then another Grade 5 bolt?

The probability of the intersection of two events.

- **Multiplication Rule**

$$P(A \cap B) = P(B|A) \cdot P(A) = P(A|B) \cdot P(B)$$

The joint probability of A and B simultaneously occuring can be written as the product of a conditional probability and an unconditional probability.

You can also switch this rule around which can be useful depending which probabilities you know...

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Example: Prosthetic Leg Tree Diagram (from last section of notes)

Let D be the event that a part is defective
Let L be the event that a part came from line 1

Find $P(D \cap L)$

e.g. (cont.)

Conditional probabilities work nicely with tree diagrams:

Example: Suppose two cards are randomly drawn from a deck of 52 cards, one-by-one, without replacement. There are 13 of each suit (diamonds, spades, hearts, clubs).

Find the probability that a diamond is drawn on the first AND on second draw.

Let A be the event that a diamond is draw on the first draw.
Let B be the event that a diamond is draw on the second draw.

$$P(A \cap B) = P(B|A) \cdot P(A)$$

$$= \frac{12}{51} \cdot \frac{13}{52}$$

$$= \frac{12}{51} \cdot \frac{1}{4} = \frac{3}{51} = 0.0588$$

- **Extension of Multiplication Rule**

$$P(A_1 \cap A_2 \cap A_3) = P(A_1 \cap (A_2 \cap A_3))$$

$$= P(A_1|A_2 \cap A_3) \cdot P(A_2 \cap A_3)$$

$$= P(A_1|A_2 \cap A_3) \cdot P(A_2|A_3) \cdot P(A_3)$$

Total Probability Rule...

Consider breaking the sample space into an A and A' or breaking the sample space up into a B and B'.

Notice that $B = (B \cap A) \cup (B \cap A')$, and these two sets are mutually exclusive. Thus, we can write

$$P(B) = P(B \cap A) + P(B \cap A').$$

By applying our multiplication rule to this equation, we get the following...

- **Total Probability Rule** (two events)
  For any events A and B,

$$P(B) = P(B \cap A) + P(B \cap A')$$

$$= P(B|A) \cdot P(A) + P(B|A') \cdot P(A')$$

In the previous situation, $S = A \cup A'$. The complements are mutually exclusive and comprise the full sample space.

If we can break-up S into k mutually exclusive sets, we can extend the previous rule...

- **Total Probability Rule** (multiple events)
  Assume $E_1, E_2, \ldots, E_k$ are k mutually exclusive and exhaustive sets. Then

$$P(B) = P(B \cap E_1) + P(B \cap E_2) + \ldots + P(B \cap E_k)$$

$$= P(B|E_1) \cdot P(E_1) + P(B|E_2) \cdot P(E_2) + \ldots + P(B|E_k) \cdot P(E_k)$$

Example: Arizona Lung Association

- According to the Arizona Chapter of the American Lung Cancer Association, 7.0% of the population has lung disease. Of those having lung disease, 90% are smokers; of those not having lung disease, 25.3% are smokers.

  Determine the probability that a person selected randomly is a smoker.

  ANS:

e.g. (cont. - tree diagram)