

## Chapter 2: PROBABILITY

### Part 3: Probability Introduction

Section 2.2 & 2.3

#### Introduction

- Probability is used to quantify the likelihood, or chance, that an outcome of a random experiment will occur.
  - “There’s a high chance that my furnace will break-down this winter.”
  - “The package says the chance that the bulb I planted will grow is 90%.”
  - “There’s a 30% chance of rain today.
- Probabilities for a random experiment are often assigned on the basis of a reasonable model of the system under study.
  - If you play a lottery where you can choose 3 digits, what is the probability that your number is drawn?
  - Under the *assumption* of **equally likely outcomes**, you have a 1/1000 chance of winning.

#### • Equally likely outcomes:

One approach is to base probability assignments on the simple concept of equally likely outcomes.

Whenever a sample space consists of  $N$  possible outcomes that are equally likely, the probability of each is  $1/N$



**Example:** Rolling a ‘fair’ die.

- The elements in the sample space  $\{1, 2, 3, 4, 5, 6\}$  are equally likely.

Probability of rolling 1 or  $P(1) = 1/6$

$$P(2) = 1/6$$

$$P(3) = 1/6$$

$$P(4) = 1/6$$

$$P(5) = 1/6$$

$$P(6) = 1/6$$

If we roll a die 60 times, the expected number of ones is 10 (of course there is some variability since this is a random experiment). If we get 0 ones, we may be concerned that this is not a ‘fair die’, it *could* happen, but not likely).

- Sometimes we're interested in an event that is composed of several outcomes from a sample space. In that case, the probability of the event occurring is the sum of the individual outcome probabilities.

**Example:** For a 'fair' die, what is the probability of rolling an even?

- $E$  is the event of rolling an even and has elements  $\{2, 4, 6\}$

$$P(E) =$$

**Example:**

- In the production of aluminum rods, specifications are given for the length and the diameter. For each rod, the length is classified as too short, too long, or OK, and the diameter is classified as too thin, too thick, or OK. In a population of 1000 rods, the number of rods in each class is as follows:

		DIAMETER		
		Too Thin	OK	Too Thick
LENGTH	Too Short			
	OK			
	Too Long			

A rod is sampled at random from this population. What is the probability that it is too short?

ANS: We can think of each of the 1000 rods as an outcome in a sample space =  $\{rod_1, \dots, rod_{1000}\}$ . Each of the 1000 outcomes is equally likely. We count the number of outcomes corresponding to the event.

- **Probability of an event** (discrete sample space)

For a discrete sample space, the *probability of an event*  $E$ , denoted as  $P(E)$ , equals the sum of the probabilities of the outcomes in  $E$ .



**Example:** Printed circuit board defects.

- An inspection for defects resulted in the following table:

Number of defects	Proportion of boards with number of defects
0	0.70
1	0.15
2	0.05
3 or more	0.10

If a board is selected at random, what is the probability that...

i) it has no defect?

ii) it has 2 or less defects?

**Example:** Sampling without replacement.

- In a set of 5 bottles, one has a small fracture. If you randomly selected a pair of bottles, what is the probability that the fractured bottle is chosen?



ANS:

Notes

### **Axioms of Probability**

Probability is a number that is assigned to each member of a collection of events from a random experiment that satisfies the following properties:

If  $S$  is the sample space and  $E$  is any event in a random experiment,

1)  $P(S) = 1$

2)  $0 \leq P(E) \leq 1$

3) For two events  $E_1$  and  $E_2$  with  $E_1 \cap E_2 = \emptyset$ ,

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

### Addition Rules of Probability

The probability of a joint event can often be determined from the probabilities of the individual events that comprise it.

- Probability of a Union

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

**Example:** Newspaper readership

- In a certain suburb, 65% of all households subscribe to the metropolitan newspaper, 55% subscribe to the local paper (obviously, some people subscribe to both, but some may subscribe to neither). Suppose, 30% subscribe to both.

If a house is selected at random, what is the probability that it subscribes to (1) at least 1 of the 2 newspapers, and (2) exactly one of the two newspapers?

ANS: Define events...

Let  $A = \{\text{household subscribes to metropolitan paper}\}$

Let  $B = \{\text{household subscribes to local paper}\}$

ANS(cont.)

$$P(A) = 0.65$$

$$P(B) = 0.55$$

$$P(A \cap B) = 0.30$$

(1)  $P(\text{household subscribes to at least one of the newspaper})$

$$= P(A \cup B)$$

$$= P(A) + P(B) - P(A \cap B)$$

$$=$$

$$=$$

(2)  $P(\text{household subscribes to exactly 1 newspaper})$

Think of the sets...

only to the metropolitan paper:  $A \cap B^c$

only to the local paper:  $B \cap A^c$

As the intersection of these two sets is empty (is the *null set*), then

$$P((A \cap B^c) \cup (B \cap A^c))$$

$$= P(A \cap B^c) + P(B \cap A^c) - 0$$

$$=$$

- Complements and the Sample Space  
If  $A$  and  $A^c$  are complementary events, then  
$$P(A) + P(A^c) = 1$$

### Mutually Exclusive Events

A collection of events  $E_1, E_2, \dots, E_k$  is said to be mutually exclusive if for all pairs,

$$E_i \cap E_j = \emptyset$$

For a collection of mutually exclusive events,

$$P(E_1 \cup E_2 \cup \dots \cup E_k) = P(E_1) + P(E_2) + \dots + P(E_k)$$

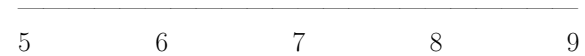
Thus, if  $A$  and  $B$  are mutually exclusive events,

$$P(A \cup B) = P(A) + P(B)$$

**Example:** Mutually exclusive events on the real number line (Example 2-21 on p.40)

- Let  $X$  denote the pH of a sample.

Consider the event that  $X$  is greater than 6.5 but less than or equal to 7.8.



$$P(6.5 < X \leq 7.8)$$

$$= P(6.5 < X \leq 7.0) + P(7.0 < X \leq 7.5) + P(7.5 < X \leq 7.8)$$

The probability is the sum of the probability of mutually exclusive events.

If these individual probabilities are easily accessible, then this is a good way to go (breaking into parts to get the total).