Chapter 2: PROBABILITY
Part 2: Counting Techniques
Section 2.1 (cont.)

Counting techniques

- Sometimes, instead of writing out all the outcomes for a sample space, we instead consider the counts of the number of outcomes for analysis

- **Multiplication Rule**
  - If an operation can be described in a sequence of k steps,
    and the number of ways of complete step 1 is n_1,
    and the number of ways of complete step 2 is n_2,
    and so forth, then

    then the total number of ways of complete the operation is

$$n_1 \times n_2 \times \cdots \times n_k$$

  - **Example**: As in the automobile maker example, there were $2 \times 2 \times 3 \times 4 = 48$ possible vehicles
  - **Example**: In the game *Guitar Hero*, you get to choose a character/guitar/venue combination. You have 8 characters to choose from, 6 guitars, and 4 venues to choose from. There are $8 \times 6 \times 4 = 192$ possible options

- **Permutations**

  - A permutation of the elements is an ordered sequence of the elements.

  - The number of permutations of n different elements is $n!$ (pronounced n factorial) where

$$n! = n \times (n-1) \times \cdots \times 2 \times 1$$

  - **Example**: Consider the set of three numbers $\{1, 2, 3\}$. There are $3! = 3 \times 2 \times 1 = 6$ permutations of the set. Here are the possible permutations of the elements in the set: (1,2,3),(1,3,2),(2,1,3),(2,3,1),(3,1,2),(3,2,1)

    There are 3 ways to 'fill' the first slot. After you choose the first, there are 2 ways to 'fill' the second slot. And then there is only 1 way left to fill the last slot.

  - **Example**: Five people stand in line at a move theater. Into how many different orders can they be arranged?
    Ans:

- **Permutations of subsets**
  - Sometimes we are interested in counting the number of permutations of subsets of a certain size chosen from a larger set.
  - **Example**: Five lifeguards are available for duty. There are three lifeguard stations. In how many ways can three lifeguards be chosen and ordered among the stations?

    Five ways to choose the first, 4 ways to choose the second, 3 ways to choose the last station.

  - In general, the number permutations of r objects chosen from a group of n objects is

$$P_r^n = n \times (n-1) \times \cdots \times (n-r+1)$$

  - Or this can be stated in factorial notation as

$$P_r^n = n(n-1)\cdots(n-r+1)$$

$$= \frac{n(n-1)\cdots(n-r+1)(n-r)(n-r-1)\cdots(3)(2)(1)}{(n-r)(n-r-1)...(3)(2)(1)}$$

$$= \frac{n!}{(n-r)!}$$

Example: Find the number of ways of selecting a president, vice president, secretary, and treasurer in a club of 15 people.

- **Permutations of Similar Objects**
  - Suppose in a set of n objects, there are n_1 indistinguishable elements of group 1, n_2 indistinguishable elements of group 2, and so forth, such that there are r groups and $n = n_1 + n_2 + \cdots n_r$. How many permutations are there of the n objects?
  - **Example**: On a leather saddle being manufactured, 2 stirrups (s) and 3 ornamental brads (o) must attached. What are the number of possible sequences of accomplishing this task?

    ANS: 10 or ssooo, sosoo, sooso, sooos, ossoo, ososo, osoos, oosso, oosos, oooss
  - In general, the number of permutations is

$$\frac{n!}{n_1! n_2! \cdots n_r!}$$

- **Combinations**

  – In many problems we are interested in the number of ways of selecting r objects from n without regard to order. These selections are called <u>combinations</u>.

  – The number of combinations, subsets of size r that can be selected from a set of n elements, is denoted as $\binom{n}{r}$ or C_r^n and pronounced "n choose r" and

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

  – **Example**: At a company with 35 engineers, the boss will be choosing 5 to go to a conference. How many different groups of 5 members are there to choose from?

    ANS:

If you're looking for the number *arrangements*, you're probably considering a permutation... order matters.

If you're choosing a subset where the order doesn't matter (like in a team of equal players or a committee), then you're probably considering a combination.