22s:039
Homework 7

Assigned Friday, October 16
Due Friday, October 23

Chapter 5: Joint continuous distributions,
Covariance, correlation, bivariate normal, linear combinations of r.v.

For each problem, provide the solution and any work that can be used for partial credit.

1. Consider the joint probability density function,

$$f_{XY}(x, y) = \frac{8}{81}xy \qquad \text{for the range of } 0 < x < 3 \text{ and } 0 < y < x$$

   Determine and provide each of the following:

   (a) $P(X < 1, Y < 2)$
   (b) $P(1 < X < 2)$
   (c) $E(X)$
   (d) $P(Y > 1)$
   (e) $E(Y)$

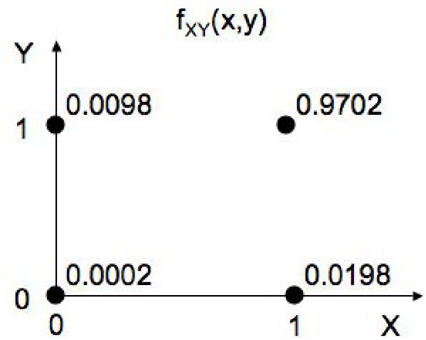
2. Suppose that random variables X and Y have the joint density function

$$f_{XY} = \frac{4}{81}xy \qquad \text{for} \qquad 0 < x < 3 \qquad \text{and} \qquad 0 < y < 3.$$

   (a) Find E(X+Y).
   (b) Find the conditional distribution of X given that Y=2, or $f_{X|Y=2}(x)$.
   (c) Find $E(X|Y = 2)$

More on back...

3. Determine the covariance and correlation for the joint probability distribution shown below (this is a discrete distribution). Be sure to include the values of E(XY), E(X), and E(Y) in your answer.



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4. Let X and Y represent concentration and viscosity of a chemical product. Suppose X and Y have a bivariate normal distribution with $\sigma_X = 4, \sigma_Y = 1, \mu_X = 2, \mu_Y = 1$ and the ρ listed below.

   For parts a, b, c below, sketch a rough contour plot (view from above) of the joint probability density function.

   a) $\rho = 0$ b) $\rho = 0.8$ c) $\rho = -0.7$

5. Suppose X and Y have a bivariate normal distribution with $\sigma_X = 0.04, \sigma_Y = 0.08$, $\mu_X = 3, \mu_Y = 7.7$ and $\rho = 0$.

   Determine the following:
   a) $P(2.95 < X < 3.05)$
   b) $P(7.60 < Y < 7.80)$
   c) $P(2.95 < X < 3.05, 7.60 < Y < 7.80)$

6. When a door is installed, it fits into the *casing* around the door (which is installed first). If the door is too wide, it won't fit into the casing.

   The width of a casing for a door is normally distributed with a mean of 24 inches and a standard deviation of 1/8 inch. The width of a door is normally distributed with a mean of 23 7/8 inches and a standard deviation of 1/16 inch. Assume independence.

   a) Determine the mean and standard deviation of the difference between the casing width and the door width.

   b) What is the probability that the door does not fit in the casing?