22s:039
Homework 2 Solutions part 2

2A-1 The proportion of people in a given community who have a certain disease is 0.005. A test is available to diagnose the disease. If a person has the disease, the probability that the test will produce a positive signal is 0.99. If a person does not have the disease, the probability that the test will produce a positive signal is 0.01.

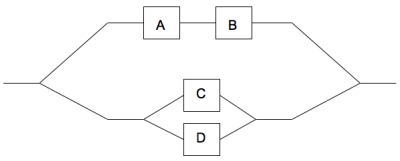
If a person tests positive, what is the probability that the person actually has the disease?

ANS:
Let D be the event of having the disease.
Let + be the event that the test shows a positive signal.
$P(D)$=0.005
$P(+|D)$=0.99
$P(+|D')$=0.01

$$P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{P(+|D)P(D)}{P(+|D)P(D) \ + \ P(+|D')P(D')} = \frac{(0.99)(0.005)}{(0.99)(0.005) \ + \ (0.01)(0.995)}$$

$$\approx 0.3322$$

2A-2 A system consists of four components connected as shown in the following diagram.



Assume A, B, C, and D function independently. If the probabilities that A, B, C, and D fail are 0.08, 0.06, 0.08, and 0.20, respectively, what is the probability that the system functions? *(Hint: work this problem in three distinct steps, each of which was discussed in class)*

ANS:

i. For the top path to function, you need both A and B working.
P(top path functions)=(0.92)(0.94)=0.8648

ii. For the bottom path to function, you need either C or D working.
P(bottom path functions)=P(C or D)=1-P(neither C nor D)=1-(0.08)(0.20)=0.984

iii. For the system to function, we need either the top or the bottom working.
P(system works)=P(top or bottom works)=1-P(both fail)=1-(0.1352)(0.016)=0.9978

2A-3 (a) 0.95

(b) 0.95