

**Instructions:**

- i. Each problem carries two points each - the exam will be graded on 20.
- ii. Italicized part of problems signify repeated information.
- iii. Good Luck and Happy Halloween!

**Problem 1** Actuaries have modeled the number of unsuccessful attempts for a typical candidate before a successful attempt in an in-house training exam. They have concluded that the number of unsuccessful attempts follows the Poisson distribution with parameter  $\Lambda$ , where  $\Lambda$  follows the gamma distribution with mean 2 and variance 3. Calculate the probability that a candidate selected at random will have no more than 1 unsuccessful attempt before passing the exam.

**Problem 2** For a population of individuals, you are given: (i) Each individual has a constant force of mortality. (ii) The forces of mortality are uniformly distributed over the interval (0,1.5). Calculate the probability that an individual drawn at random from this population dies within one year.

**Problem 3**  $X$  is a random variable taking values in the interval (0,2) with unit mean. Its density on the interval (0,1) is proportional to that of a Beta with parameters  $\alpha = 2$  and  $\beta = 1$ . On the interval (1,2) the density is constant. Find  $\text{Var}(X)$ .

**Problem 4** The unlimited severity distribution for claim amounts under an auto liability insurance policy is given by the cumulative distribution:

$$F(x) = 1 - 0.75 \exp\{-0.02x\} - 0.25 \exp\{-0.001x\}, \quad x \geq 0$$

The insurance policy pays amounts up to a limit of 1000 per claim. Calculate the variance of the payment under this policy for one claim.

**Problem 5** The number of yards per game that a certain varsity football team concedes (to the opponent) were previously modeled by a two-parameter Pareto distribution with  $\alpha = 2$  and  $\theta = 350$ . The athletic director worried about its defense, decides to reward the assistant coach in-charge of defense by paying a bonus of \$50 for each yard the opponent scores below 350. No bonus is paid if total yards exceed 350. Calculate the expected bonus per game that will accrue to the assistant coach assuming that the team continues to play its usual defense.

**Problem 6** The number of claims in a period has a geometric distribution with mean 5. The amount of each claim  $X$  follows discrete uniform on the set  $\{0, 1, 2, 3, 4\}$ . The number of claims and the claim amounts are independent.  $S$  is the aggregate claim amount in the period. Calculate  $F_S(3)$ .

**Problem 7** *The number of claims in a period has a geometric distribution with mean 5. The amount of each claim  $X$  follows discrete uniform on the set  $\{0, 1, 2, 3, 4\}$ . The number of claims and the claim amounts are independent.  $S$  is the aggregate claim amount in the period. Approximate  $F_S(3)$  using the normal approximation for  $S$ .*

**Problem 8** The number of claims assumes values 0, 1, 2 and 3 with probabilities 0.5, 0.3, 0.15 and 0.05, respectively. The claim distribution is uniform on the set  $\{1, 2, 3\}$ . Assuming that the number of claims and the amounts of the claims are independent of each other, find the probability that the aggregate claims is exactly equal to 4.

**Problem 9** Due to the spiralling cost of health care in Agingvalley, a small business has decided to introduce coverage modifications in its health care coverage for its employees. First, it decides to

not cover certain illness - the thought being that occurrences of these, predominantly, can be self-controlled. The first measure is estimated to reduce the loss frequency by 25%. Second, it decides to introduce a deductible of \$100 per loss. The loss has an exponential distribution with mean of 200 in the year 2004. Find the percentage reduction in the health costs due to the above two modifications in the year 2004.

**Problem 10** *Due to the spiralling cost of health care in Agingvalley, a small business has decided to introduce coverage modifications in its health care coverage for its employees. First, it decides to not cover certain illness - the thought being that occurrences of these, predominantly, can be self-controlled. The first measure is estimated to reduce the loss frequency by 25%. Second, it decides to introduce a deductible of \$100 per loss. The loss has an exponential distribution with mean of 200 in the year 2004. The inflation in the loss amounts between the year 2004 and 2005 is estimated to be 12%. The business owner decides to maintain the same percentage reduction in the health costs due to coverage modifications in the year 2005 as in year 2004, but by introducing **in addition** (to the reduction in covered illness and deductible) a policy limit in 2005. Find the policy limit.*

**Problem 11** Faunaville Natural Resources Department introduce live full grown fish to the Faunaville Lake at a rate of 100 per day. Citizens of the community arrive at a Poisson rate of 25 per day to the shores of the lake for fishing, with the size of their catch following a Binomial distribution with  $n = 4$  and  $p = 0.75$ . Find the probability that the population of the fish in the lake will ever dip below the current level.

**Problem 12** *Faunaville Natural Resources Department introduce live full grown fish to the Faunaville Lake at a rate of 100 per day. Citizens of the community arrive at a Poisson rate of 25 per day to the shores of the lake for fishing, with the size of their catch following a Binomial distribution with  $n = 4$  and  $p = 0.75$ . Given that the population of the fish in the lake did fall below the current level, find the probability that it decreased by more than 2 the first time it fell below the current level.*