Exam III Solution.

Problem 1.

$$X_i \begin{cases} 50 & \text{w.p} \quad 1/2 \\ 150 & \text{w.p} \quad 1/2 \end{cases}$$

$$\text{Account balance at } t = u + \sum_{i=1}^{N(t)} X_i - 105t$$

where $N(t) \sim \text{Poisson}(1/\text{day})$

$$\Pr\left(u + \sum_{i=1}^{N(t)} X_i - 105t > u \text{ for some } t \right)$$

$$= \Pr\left(105t - \sum_{i=1}^{N(t)} X_i < 0 \text{ for some } t \right)$$

$$= \psi(0) = \frac{1}{1+\theta}.$$

Since $(1+\theta)\underbrace{\lambda}_{=1}\underbrace{p_1}_{=100} = 105$, $\quad \frac{1}{1+\theta} = \frac{100}{105} = 0.952380952$

Problem 2. See Exercise 13.12 & (13.5.4)

$$M_{L_1}(r) = \frac{1}{p_1 r}\left[M_X(r) - 1 \right]$$

$$= \frac{1}{p_1 r}\left[1 + p_1 r + p_2 \frac{r^2}{2!} + p_3 \frac{r^3}{3!} + \cdots - 1 \right]$$

$$= 1 + \frac{1}{2}\cdot\frac{p_2}{p_1} r + \frac{1}{6}\cdot\frac{p_3}{p_1} r^2 + \cdots$$

$$\therefore \; EL_1 = M_{L_1}'(0) = \frac{p_2}{2p_1} = \frac{\frac{1}{2}(50^2 + 150^2)}{2(100)} = 62.5$$

Problem 3. Adjustment Coefficient $R = 0.2$ satisfy

$$1 + (1+\theta)p_1 r = M_X(r) \quad \& \quad (1+\theta)p_1\lambda = c$$

$$\therefore \quad 1 + \frac{c}{\lambda}(0.2) = M_X(0.2)$$

Since $M_X(0.2) = \frac{1}{5}\left(e^{0.2} + e^{0.4} + \cdots + e\right) = 1.895833803$,

$$c = \frac{0.895833803}{0.2} \times 3 = 13.43750704$$

Problem 4 See Example 13.6.1

$$\psi(u) = \frac{1}{1+\theta} e^{-\frac{\theta}{(1+\theta)}\frac{u}{p_1}}$$

where $p_1 = 7.5$

$$u = \frac{1}{4}(\lambda p_1) = \frac{1}{4}(750) = 187.5$$

average day's consumption

$$(1+\theta)\lambda p_1 = 800 \quad \therefore \quad 1+\theta = \frac{800}{750}.$$

$$\frac{1}{1+\theta} = \frac{750}{800} \quad \frac{\theta}{1+\theta} = \frac{50}{800}$$

$$\therefore \quad \psi(187.5) = \frac{750}{800} e^{-\left(\frac{50}{800}\right)\left(\frac{100}{4}\right)}$$

$$= 0.196510675$$

Problem 5

$\psi(u) = 0.1\exp(-3u) + x\exp(-yu)$

$\theta = \frac{3}{7}$ $\quad R = 1.5$

$\psi(0) = \frac{1}{1+\theta} \Rightarrow \frac{1}{1+\frac{3}{7}} = 0.7 = 0.1 + x$

$\therefore x = 0.6$

From (13.6.13) & Figure 13.6.2,

$1.5 = \min(3, y)$

$\therefore y = 1.5$

Problem 6

$u = 10$ (Billion)

$c = 500$ / yr

$\lambda = 250$ / yr

$X \sim$ exponential w/ mean $= 1.85$

$\Pr\left(U(t) = u + ct - \sum_{i=1}^{N(t)} X_i < -10 \text{ for some } t \right)$

$= \Pr\left(20 + 500t - \sum X_i < 0 \text{ for some } t \right)$

$= \psi(20)$

$(1+\theta)p_1\lambda = c$

$= \frac{1}{1+\theta} e^{-\frac{\theta}{1+\theta}p_1 \cdot 20}$

$1+\theta = \frac{500}{1.85 \cdot 250} = \frac{40}{37}$

$= \frac{37}{40} e^{-\frac{3}{40}\cdot\frac{1}{1.85}\cdot 20}$

$= 0.411160203$