Problem 1

$N \rightarrow$ # of Auto Windshield Claims

$N / \ell \sim Poi(\ell)$

$\ell \sim$ Gamma (Mean $= 4$, Variance $= 4$).

For a Gamma Mean $= \alpha\beta = 4$

Variance $= \alpha\beta^2 = 4$

$\boxed{\text{Course 3 Tables Gamma Notation is } (\alpha, \theta)}$

Hence $\beta = 1$ and $\alpha = 4$.

Hence the $\underline{\text{marginal}}$ distbn. of N

will be $NB(r = \alpha = 4, \beta = 1)$.

Using the above and the tables we have

$Pr(N \leq 2) = Pr(N=0) + Pr(N=1) + Pr(N=2)$

$= \left(\frac{1}{2}\right)^4 + \frac{(4)}{2^5} + \frac{4 * 5}{2!\ 2^6}$

$= \frac{1}{16} + \frac{1}{8} + \frac{5}{32} = \underline{\underline{\frac{11}{32}}}$ // $[\simeq 0.34375]$.

Problem 2 $E(N) = 100$ $Var(N) = 750$

$E(X) = 1100$ $Var(X) = (70)^2 = 4900$

$Pr(S \leq 150{,}000) = ?$

$E(S) = EN * EX = 110{,}000$

$Var(S) = E(N) * Var(X) + (EX)^2 \, Var(N)$

$= 100 * 4900 + (1100)^2 * 750$

$\Rightarrow \sigma_S = 100 * \sqrt{49 + 121 * 750} = 100 * 301.3287$

$= 30{,}132.87.$

Hence $Pr(S \leq 150{,}000)$

$= Pr\left(\dfrac{S - 110{,}000}{30{,}132.87} \leq \dfrac{150{,}000 - 110{,}000}{30{,}132.87} \right)$

$\simeq \Phi(1.327454) = 0.907821$

Problem 3

X is a constant 50

N ~ Geometric with mean 4.

$$E\left[(S-125)_+\right] = E\left[(50N-125)_+\right]$$

$$= 50\ E[N-2.5]_+$$

$$= 50\left[E\left[[N-2.5] \mid N \geq 3\right] Pr(N \geq 3)\right]$$

$$= 50\ Pr(N \geq 3)\left[E\left(N-3+0.5 \mid N \geq 3\right)\right]$$

$$= 50\ Pr(N \geq 3)\left[0.5 + E(N-3 \mid N \geq 3)\right]$$

Memoryless Property of geometric.

$$= 50 * \left[1 - \frac{1}{5} - \frac{4}{25} - \frac{16}{125}\right]\left[0.5 + 4\right]$$

$$= 50 * \frac{[125-25-20-16]}{125} * 4.5 = \frac{128}{5} * 4.5 = \underline{\underline{115.2}}$$

Problem 4

$N \sim DU\{1, \dots 6\}$.

$EN = 3.5$

$Var(N) = \frac{6^2-1}{12} = \frac{35}{12}$

$X \stackrel{d}{=} N$

$EX = 3.5$

$Var(X) = \frac{35}{12} = 3 - \frac{1}{12}$ //

$ES = (3.5)^2 = \underline{12.25}$

$Var(S) = EN\,Var(X) + (EX)^2\,Var(N)$

$= (3.5) * \frac{35}{12} + (3.5)^2 \frac{35}{12} = \underline{45.9375}$

Let Y_i be the winnings of the player in the ith game.

Then the players networth after 1000 games will be $\sum_1^{1000} Y_i - 12.5 * 1000 + 10,000$

Hence $Pr\left(\sum_1^{1000} Y_i - 12.5 * 1000 + 10,000 \geq 10,000\right)$

$= Pr\left(\sum_1^{1000} Y_i - 12.25 * 1000 \geq 250\right)$

$= Pr\left(\frac{\sum_1^{1000} Y_i - 12.25 * 1000}{\sqrt{45.9375 * 1000}} \geq \frac{250}{\sqrt{45,937.5}}\right)$

$\approx 1 - \Phi(1.166424) \simeq 0.121722$

Problem 5

$$F_S(4) = Pr(S=0) + Pr(S=1) + Pr(S=2) + Pr(S=3) + Pr(S=4)$$

$S=0$ iff $N=0$ hence $Pr(S=0) = Pr(N=0) = 1/5$

$S=1$ iff $N=1$ and $X_1=1 \Rightarrow Pr(S=1) = Pr(N=1)\,Pr(X_1=1)$

$$= \left(\frac{4}{5} * \frac{1}{5}\right) * \left(\frac{1}{4}\right)$$

$$= \underline{\underline{1/25}}$$

$\{S=2\} = \{N=1 \text{ and } X_1=2\} \cup \{N=2 \text{ and } X_1=1=X_2\}$

$$\Rightarrow Pr(S=2) = \left(\frac{4}{5} * \frac{1}{5}\right) * \frac{1}{4} + \left(\frac{4}{5}\right)^2 * \frac{1}{5} * \left(\frac{1}{4}\right)^2$$

$$= \frac{1}{25} + \frac{1}{125} = \underline{\underline{\frac{6}{125}}} //$$

$\{S=3\} = \{N=1 \text{ and } X_1=3\} \cup \{N=2 \text{ and either } X_1=1 \text{ and } X_2=2 \text{ or } X_1=2 \text{ and } X_2=1\}$

$\cup \{N=3 \text{ and } X_1=1=X_2=X_3\}.$

$$\Rightarrow Pr(S=3) = \frac{4}{5} * \frac{1}{5} * \frac{1}{4} + \left(\frac{4}{5}\right)^2 * \left(\frac{1}{5}\right) * \left(2 * \frac{1}{4} * \frac{1}{4}\right)$$

$$+ \left(\frac{4}{5}\right)^3 \left(\frac{1}{5}\right) * \left(\frac{1}{4}\right)^3 = \frac{1}{25} + \frac{2}{125} + \frac{1}{625} = \frac{36}{625}$$

Hence $F_S(3) = \frac{1}{5} + \frac{1}{25} + \frac{6}{125} + \frac{36}{625} = \frac{125+25+30+36}{625} = \underline{\underline{\frac{216}{625}}} //$

Similarly,

$$Pr(S=4) = \frac{4}{5} * \frac{1}{5} * \frac{1}{4} + \left(\frac{4}{5}\right)^2 * \left(\frac{1}{5}\right) * \left(\frac{1}{4}\right)^2 [1+2]$$

$$+ \left(\frac{4}{5}\right)^3 * \frac{1}{5} * \left(\frac{1}{4}\right)^3 * 3 \quad + \left(\frac{4}{5}\right)^4 * \frac{1}{5} * \left(\frac{1}{4}\right)^4$$

$$= \frac{1}{25} + \frac{3}{125} + \frac{3}{625} + \frac{1}{3125} = \frac{125+75+15+1}{3125} = \frac{216}{3125}$$

or $\quad F_S(4) = F_S(3) + Pr(S=4) = \frac{216}{625} + \frac{216}{3125} = \frac{1296}{3125}$

$$\simeq \underline{\underline{0.41472}}$$

Problem 6

Hunt's Bonus =

losses.

$$800{,}000 * 0.15 * \left(0.65 - \frac{X}{800{,}000}\right)_+$$

earned premium.

acceptable lossratio

Hunt's share in the profits.

$$E(\text{Hunt's bonus})$$

$$= 0.15\, E\left[\left(520{,}000 - X\right)_+\right].$$

$$(a-x)_+ = \begin{cases} a-x & x < a \\ 0 & x > a \end{cases} \qquad x \wedge a = \begin{cases} x & x < a \\ a & x > a \end{cases}$$

Hence $(a-x)_+ = a - x\wedge a$

Hence

$$0.15\, E\left(\left(520{,}000 - X\right)_+\right) = 0.15\left[520{,}000 - E(X \wedge 520{,}000)\right]$$

$$= 0.15\left[520{,}000 - \frac{500{,}000}{2-1}\left[1 - \left(\frac{500{,}000}{500{,}000+520{,}000}\right)^{2-1}\right]\right]$$

$$= 39{,}764.71 \;//$$