

Some problems on Splines.

① It is known that

$$f(x) = 1.5x - 0.5x^3.$$

Find the coefficients in the expansion of $f(\cdot)$ in terms of the basis $\{1, (x+1), (x+1)^2, (x+1)^3\}$.

i.e. find a, b, c and d \Rightarrow

$$f(x) = a + b(x+1) + c(x+1)^2 + d(x+1)^3$$

② It is known that

$$f(x) = \begin{cases} x & -4 \leq x < 0 \\ x^3 + x & 0 \leq x \leq 1 \\ a + bx + cx^2 & 1 \leq x \leq 9 \end{cases}$$

is a ~~sp~~ cubic spline. Find a, b and c .

③ Consider

$$f(x) = \begin{cases} 28 + 25x + 9x^2 + x^3 & -3 \leq x \leq -1 \\ 26 + 19x + 3x^2 - x^3 & -1 \leq x \leq 0 \\ 26 + 19x + 3x^2 - 2x^3 & 0 \leq x \leq 3 \\ -163 + 208x - 60x^2 + 5x^3 & 3 \leq x \leq 4 \end{cases}$$

Which of the five endpoint conditions could have been used in ~~to~~ developing this spline.

④. Find the coefficients in the Lagrange's formula for the polynomial ~~curve~~ of degree 2 which passes through the points $(2, 50)$, $(4, 25)$ and $(5, 20)$.