Sample Exam

(1) Jim is considering different 2 types of insurances to cover against a random loss X, which is distributed as an Exponential with mean 100. His utility fn. is given by

$$U(w) = -e^{-0.001w} \quad \forall w \geq 0.$$

(a) For an ~~simple~~ insurance with deductible d, i.e.

$$I_d(X) = (X-d)_+$$

find the deductible which makes the pure premium equal to \$10.

(b) For a proportional insurance with proportion k, find k which makes the pure premium equal to \$10.

(c) For an insurance with an ordinary deductible of d and reimbursing only 75% of the loss in excess of the deductible, find d which makes the pure premium equal to \$10.

(d) Which ~~one~~ of the two insurances in part (b) or (c) would be preferred by Jim.

ⓐ $E(X-d)_+ = \int_0^\infty (x-d)_+ \; 0.01 \, e^{-0.01x} \, dx$

$$= \int_d^\infty \underbrace{(x-d)}_{y} \; 0.01 \, e^{-0.01\overbrace{(x-d)}^{y}} \, e^{-0.01d} \, dx$$

$$= e^{-0.01d} \underbrace{\int_0^\infty y \; 0.01 \, e^{-0.01y} \, dy}_{\text{Expectation}}$$

$$= e^{-0.01d} * 100$$

Hence the deductible for a pure premium of \$10 will be given by

$$e^{-0.01d} * 100 = 10$$

$$-0.01d = \ln 0.1$$

$$d = \$230.2585$$

ⓑ $EkX = kEX = k100$

Hence the proportion in the proportional insurance corresponding to a pure premium of \$10 will be

$$k * 100 = 10$$

or $\underline{\underline{k = 0.1}}$ or $\underline{\underline{10\%}}$

(c)

$$I_d^*(X) = \begin{cases} 0 & X < d \\ 0.75(X-d) & X > d. \end{cases}$$

$E\, I_d^*(X)$

$$= \int 0.75\,(X-d)_+ \; 0.01\, e^{-0.01x}\, dx$$

$$= \int_d^\infty 0.75\,(X-d)\; 0.01\, e^{-0.01x}\, dx$$

$$= \int_d^\infty 0.75\,(x-d)\; 0.01\; e^{-0.01(x-d)}\, e^{-0.01d}\, dx$$

$$= 0.75\, e^{-0.01d} \underbrace{\int_0^\infty y\; 0.01\; e^{-0.01y}\, dy}_{\text{expectation}}$$

$$= 0.75\, e^{-0.01d}\; 100$$

$$= 75 * e^{-0.01d}.$$

Hence the deductible in this case is given by

$$75 * e^{-0.01d} = 10$$

or $d = \$\underline{201.49}$

(d) Since it is an exponential utility function, the initial wealth is unimportant, or assume $W_0 = 10$.

So the final wealth in the case of proportional insurance will be

$$10 - X + 0.1X - \underset{\substack{\uparrow \\ \text{Premium}}}{10}$$

and in the case of the deductible + coinsurance it will be

$$10 - X + 0.75(x-d)_+ - 10$$

Hence the expected utility of the future wealth under the proportional insurance will be

$$E(U(10 - X + 0.1X - 10)) = E(U(-0.9X))$$

$$= -\int_0^\infty e^{-0.001(-0.9x)} \, 0.01 \, e^{-0.01x} \, dx$$

$$= -0.01 \int_0^\infty e^{-(0.01 - 0.0009)x} \, dx$$

$$= \frac{-0.01}{0.01 - 0.0009} = \frac{-1}{1 - 0.09} = \frac{-1}{0.91} = -1.0989$$

The same under the second insurance will be

$$E\left(U\left(-X+0.75(X-d)_+\right)\right).$$

$$= \int_0^\infty -e^{-0.001\left(-X+0.75(X-d)_+\right)}\, 0.01 e^{-0.01x}\,dx$$

$$= \int_0^d -e^{-0.001(-x)}\, 0.01 e^{-0.01x}\,dx + \int_d^\infty -e^{-0.001(-x+0.75(x-d))}\, 0.01 e^{-0.01x}\,dx$$

$$= \underbrace{\int_0^d -0.01\, e^{-(0.01-0.001)x}\,dx}_{\text{distbn. fn. of exponential}} + \underbrace{\int_d^\infty -e^{-0.00075d}\, 0.01\, e^{-(0.01-0.00025)x}\,dx}_{\text{Survival function}}$$

$$= -\left[\frac{0.01}{0.01-0.001}\left[1-e^{-(0.009)d}\right] + \frac{e^{-0.00075d}\, 0.01\, e^{-0.00975d}}{.00975}\right]$$

$$-\left[0.9298 + 0.12364\right] = -1.053 > -1.0989$$

Hence the insurance with the deductible and coinsurance will be ~~better~~ preferred ~~than~~ to the proportional insurance.