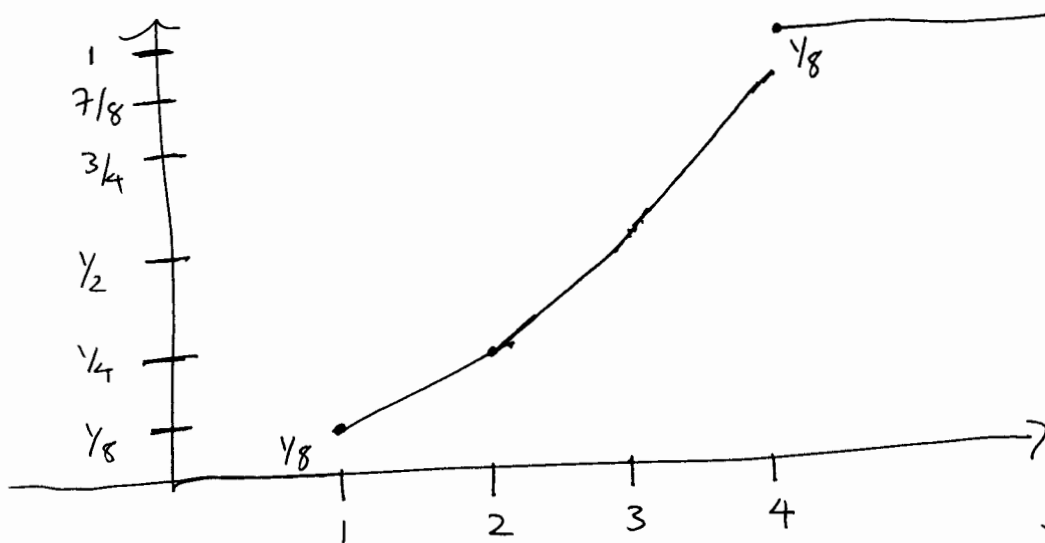


**Problem 1** For the following distribution function, finds its discrete and continuous constituents. Using the components, find the mean and variance of the distribution function. Show your steps clearly.

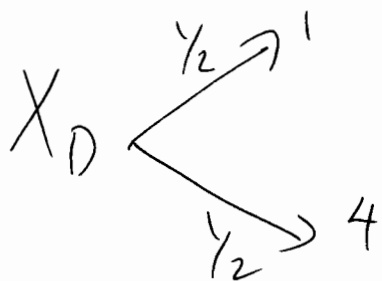
$$F(x) = \begin{cases} 0 & x < 1; \\ \frac{3ix - 1.5i(i+1) + 3}{24} & i \leq x < i+1, \quad i = 1, 2, 3; \\ 1 & \text{otherwise.} \end{cases}$$



Jump Points  $\rightarrow \{1, 4\}$ .  $P_D = Pr(X=1) + Pr(X=4) = \frac{1}{4}$

Hence  $Pr(X_D=1) = \frac{Pr(X=1)}{Pr(X=1) + Pr(X=4)} = \frac{1/8}{1/8 + 1/8} = \frac{1}{2}$

and  $Pr(X_D=4) = 1 - Pr(X_D=1) = \frac{1}{2}$



$$E X_D = 2.5$$

$$Var(X_D) = \frac{9(4-1)^2}{4} = \frac{9}{4} = 2.25$$

$$F_c(x) = \begin{cases} 0 & x < 1 \\ \frac{x-1}{6} & 1 \leq x < 2 \\ \frac{x}{3} - \frac{1}{2} & 2 \leq x < 3 \\ \frac{x}{2} - 1 & 3 \leq x < 4 \\ 0 & x \geq 4 \end{cases}$$

$$\text{or } f_c(x) = \begin{cases} \frac{1}{6} & 1 \leq x < 2 \\ \frac{1}{3} & 2 \leq x < 3 \\ \frac{1}{2} & 3 \leq x < 4 \\ 0 & \text{otherwise} \end{cases}$$

$$EX_c = \frac{1}{6} * \frac{3}{2} + \frac{1}{3} * \frac{5}{2} + \frac{1}{2} * \frac{7}{2}$$

$$= \frac{1}{4} + \frac{5}{6} + \frac{7}{4} = \frac{3+10+21}{12} = \frac{34}{12} = \underline{\underline{\frac{17}{6}}} \quad (3 - \frac{1}{6})$$

$$EX_c^2 = \frac{1}{6} * \frac{7}{3} + \frac{1}{3} * \frac{19}{3} + \frac{1}{2} * \frac{37}{3}$$

$$= \frac{7}{18} + \frac{19}{9} + \frac{37}{6} = \frac{7+38+111}{18} = \frac{156}{18}$$

$$= \frac{78}{9} = \frac{26}{3} = \underline{\underline{9 - \frac{1}{3}}}$$

$$\begin{aligned}
 \text{Var}(X_c) &= \left(9 - \frac{1}{3}\right) - \left(3 - \frac{1}{6}\right)^2 \\
 &= \cancel{9} - \frac{1}{3} - \left[\cancel{9} + \frac{1}{36} - 1\right] \\
 &= 1 - \frac{1}{3} - \frac{1}{36} = \frac{36 - 12 - 1}{36} = \underline{\underline{\frac{23}{36}}}
 \end{aligned}$$

$$\begin{aligned}
 EX &= P_D EX_D + P_C EX_C \\
 &= \frac{1}{4} * 2.5 + \frac{3}{4} * \left(3 - \frac{1}{6}\right) \\
 &= \frac{1}{4} \left[2.5 + 9 - \frac{1}{2}\right] = \frac{11}{4} = \underline{\underline{2.75}}
 \end{aligned}$$

$$\begin{aligned}
 \text{Var}(X) &= P_D P_C (EX_D - EX_C)^2 \\
 &\quad + \left[ P_D \text{Var}(X_D) + P_C \text{Var}(X_C) \right] \\
 &= \frac{3}{16} \left( \underbrace{3 - \frac{1}{6} - 2.5}_{\frac{1}{3}} \right)^2 + \left[ \frac{1}{4} * \frac{9}{4} + \frac{3}{4} * \frac{23}{36} \right] \\
 &= \frac{\cancel{3}}{16} * \frac{1}{9} + \frac{9}{16} + \frac{23}{48} \\
 &= \frac{1}{48} + \frac{27}{48} + \frac{23}{48} = \frac{51}{48} = 1 + \frac{1}{16} = \underline{\underline{1.0625}}
 \end{aligned}$$