Probabilistic Model for Life Contingencies

Shyamal Kumar

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1 Preface

In this note s and t are non-negative reals, $u \in [0, 1]$ and m and n are non-negative integers unless specified otherwise.

Without doubt the note contains many redundant formulae - in the sense that they can be easily derived from the others. To help identify a minimal subset, we have placed a blue tick mark in the background of formulae which are the constituents of one such subset. As an exercise you may try to derive the ones without such a mark from those which have such a mark.

This is the first version of a work in progress. Any comments will be appreciated - Please e-mail them to shyamal-kumar@uiowa.edu

Shyamal Kumar
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2 Survival Function and Hazard Rate

In elementary courses on probability it is customary to characterize a distribution by its distribution function or by a density function (probability mass function)in the case of continuous (discrete) distribution. In the following, always $F(\cdot)$ and $f(\cdot)$ will denote a distribution function and a density function, respectively. In Life Contingencies it will be convenient sometimes to work with $1 - F(\cdot)$ in place of $F(\cdot)$ and this is defined to be the survival function. We shall denote a survival function by $S(\cdot)$ and

$$S(\cdot) = 1 - F(\cdot)$$

The hazard rate function denoted by $\mu(\cdot)$, only for continuous distributions, is defined as

$$\mu(\cdot) = \frac{f(\cdot)}{S(\cdot)}$$

For a continuous random non-negative variable, the following table summarizes the important properties of the above quantities; the second table below gives their inter-relationships.

$F(\cdot)$	$S(\cdot)$	$f(\cdot)$	$\mu(\cdot)$
Non-decreasing	Non-increasing	Non-negative	Non-negative
$F(0) = 0$	$S(0) = 1$	-	-
$F(\infty) = 1$	$S(\infty) = 0$	$\int_0^\infty f = 1$	$\int_0^\infty \mu = \infty$
Continuous	Continuous	Existence	Existence

	$F(x) =$	$S(x) =$	$f(x) =$	$\mu(x) =$
$F(x)$		$1 - F(x)$	$\frac{d}{dx}F(x)$	$-\frac{d}{dx}\ln(1 - F(x))$
$S(x)$	$1 - S(x)$		$-\frac{d}{dx}S(x)$	$-\frac{d}{dx}\ln(S(x))$
$f(x)$	$\int_0^x f(y)dy$	$\int_x^\infty f(y)dy$		$\frac{f(x)}{\int_x^\infty f(y)dy}$
$\mu(x)$	$1 - \exp\left\{-\int_0^x \mu(y)dy\right\}$	$\exp\left\{-\int_0^x \mu(y)dy\right\}$	$\exp\left\{-\int_0^x \mu(y)dy\right\}\mu(x)$	

3 T(x) - The Protagonist of Life Contingencies

In Life Contingencies, X will denote the future lifetime of a **newborn** or equivalently the age-at-death of a newborn. For x a non-negative real number, the **notation** (x) will denote a life aged x. Now, for a life aged x, i.e. (x), $T(x)$ will denote it's future lifetime. Note that $T(0)$ and X are the very same random variables. In the following we shall assume that X is a continuous random variable.

The key assumption or definition, whichever way you see it, is

$$T(x) \overset{\mathrm{d}}{=} X - x | X > x \quad \Leftrightarrow \quad \Pr\left(T(x) \le t\right) = \Pr\left(X - x \le t | X > x\right), \quad \forall t \ge 0.$$

An implication and a generalization of the above is the Aggregate Law of Mortality (ALoM) which states,

$$T(x+y) \overset{\mathrm{d}}{=} T(x) - y | T(x) > y \quad \Leftrightarrow \quad \Pr\left(T(x+y) \le t\right) = \Pr\left(T(x) - y \le t | T(x) > y\right), \quad \forall t \ge 0.$$

The following is an explanation of the ALoM. Consider Tom, Dick and Harry aged 0, 10 and 30 respectively - according to the connotation of their names, we assume that they are completely unknown characters to the person making the following statements. The statement that Tom will have the same future lifetime distribution as Dick when Tom is aged 10 is saying $T(10) \overset{\mathrm{d}}{=} X - 10 | X > 10$. The statement that Dick will have the same future lifetime distribution as Harry when Dick is aged 30 is saying $T(30) \overset{\mathrm{d}}{=} T(10) - 20 | T(10) > 20$.

The curtate future life time of (x) is denoted by $K(x)$ and is defined as the integer part of $T(x)$ or $K(x) = \lfloor T(x) \rfloor$. Hence $K(x) \le T(x) < K(x) + 1$. Whenever we will consider the curtate future life time of (x), i.e. $K(x)$, we shall assume that x is a non-negative integer. The following table summarizes the inter-relationship between X and $T(x)$ and $K(x)$.

	$T(x)$	$K(x)$
F_X	$\mathrm{F}_{T(x)}(t) = \frac{\mathrm{F}_X(x+t) - \mathrm{F}_X(x)}{1 - \mathrm{F}_X(x)}$	$\Pr\left(K(x) \le n\right) = \Pr\left(K(x) < n+1\right) = \mathrm{F}_{T(x)}(n+1)$
S_X	$\mathrm{S}_{T(x)}(t) = \frac{\mathrm{S}_X(x+t)}{\mathrm{S}_X(x)}$	$\Pr\left(K(x) > n\right) = \Pr\left(K(x) \ge n+1\right) = \mathrm{S}_{T(x)}(n+1)$
f_X	$f_{T(x)}(t) = \frac{f_X(x+t)}{\int_x^\infty f_X(y)\mathrm{d}y}$	-
μ_X	$\mu_{T(x)}(t) = \mu_X(x+t)$	-

> There are two distribution for X which are commonly used in the SOA exams - Uniform and Exponential. It is very important that you are aware that if $X \sim U(0, \omega)$ then $T(x) \sim U(0, \omega - x)$ and instead if $X \sim Exp(\text{with mean } \lambda)$ then $T(x) \sim Exp(\text{with mean } \lambda)$. The latter is known as the **memory-less property** of the exponential.

Now we are ready to define probably the most widely used notation in Actuarial Science - ${}_tq_x$. The probability that Harry, aged 30 (unfortunately) dies **before** reaching the age of 50 (30 + 20) is given by ${}_{20}q_{30}$. In general, the probability that (x) dies before reaching the age of $x + t$ is given by ${}_tq_x$. The *counterpart* of ${}_tq_x$ is denoted by ${}_tp_x$ - ${}_tp_x = 1 - {}_tq_x$. The probability that Harry, aged 30 lives (expectedly) beyond the age of 50 (30 + 20) is given by ${}_{20}p_{30}$. A generalization of ${}_tq_x$ is given by ${}_{s|t}q_x$. The probability that Harry, aged 30 lives beyond the age of 50 (30 + 20) but dies before reaching age 75 $(30+20+25)$ is given by ${}_{20|25}q_{30}$ - this is same as saying that Harry is alive at 50 but is not alive (i.e. dead) at age 75 or equivalently saying that Harry is dead by age 75 but is not dead (i.e. alive) at age 50. The latter rephrasing yields two different formulae in the table below. In general, the probability that (x) lives to the age of $x + s$ and then dies before reaching the age of $x + s + t$ is given by ${}_{s|t}q_x$. When $t = 1$, the value of t is suppressed - in other words the **default** value for t is 1. Hence $q_x = {}_1q_x$, $p_x = {}_1p_x$ and ${}_{s|}q_x = {}_{s|1}q_x$. The following table give the inter-relationships between these quantities and the following connects them to probabilistic quantities relating to $T(x)$ and $K(x)$.

	${}_tp_x =$	${}_tq_x =$	${}_{s\|t}q_x =$
${}_tp_x$		$1 - {}_tp_x$	${}_sp_x(1 - {}_tp_{x+s}) = {}_sp_x - {}_{s+t}p_x$
${}_tq_x$	$1 - {}_tq_x$		$(1 - {}_sq_x){}_tq_{x+s} = {}_{s+t}q_x - {}_sq_x$
${}_{s\|t}q_x$	$1 - {}_{0\|t}q_x$	${}_{0\|t}q_x$	

	$T(x)$	$K(x)$
${}_tp_x$	$S_{T(x)}(t)$	$\Pr(K(x) < n) = {}_nq_x$
${}_tq_x$	$F_{T(x)}(t)$	$\Pr(K(x) \geq n) = {}_np_x$
${}_{s\|t}q_x$	$\Pr(s < T(x) \leq s + t)$	$\Pr(K(x) = m) = {}_{m\|}q_x$

4 Mortality Table & the Uniform Assumption

The following is the 1980 US CSO Female Age Nearest - CSO stands for the commissioners standard ordinary. The choice of the female table is driven by the fact that it is the one granting a longer life. We shall in the note and the course use this as our default aggregate mortality table.

x	q_x	x	q_x	x	q_x	x	q_x	x	q_x
0	0.00289	20	0.00105	40	0.00242	60	0.00947	80	0.06599
1	0.00087	21	0.00107	41	0.00264	61	0.01013	81	0.0736
2	0.00081	22	0.00109	42	0.00287	62	0.01096	82	0.0824
3	0.00079	23	0.00111	43	0.00309	63	0.01202	83	0.09253
4	0.00077	24	0.00114	44	0.00332	64	0.01325	84	0.10381
5	0.00076	25	0.00116	45	0.00356	65	0.01459	85	0.1161
6	0.00073	26	0.00119	46	0.0038	66	0.016	86	0.12929
7	0.00072	27	0.00122	47	0.00405	67	0.01743	87	0.14332
8	0.0007	28	0.00126	48	0.00433	68	0.01884	88	0.15818
9	0.00069	29	0.0013	49	0.00463	69	0.02036	89	0.17394
10	0.00068	30	0.00135	50	0.00496	70	0.02211	90	0.19075
11	0.00069	31	0.0014	51	0.00531	71	0.02423	91	0.20887
12	0.00072	32	0.00145	52	0.0057	72	0.02687	92	0.22881
13	0.00075	33	0.0015	53	0.00615	73	0.03011	93	0.25151
14	0.0008	34	0.00158	54	0.00661	74	0.03393	94	0.27931
15	0.00085	35	0.00165	55	0.00709	75	0.03824	95	0.31732
16	0.0009	36	0.00176	56	0.00757	76	0.04297	96	0.37574
17	0.00095	37	0.00189	57	0.00803	77	0.04804	97	0.47497
18	0.00098	38	0.00204	58	0.00847	78	0.05345	98	0.65585
19	0.00102	39	0.00222	59	0.00894	79	0.05935	99	1

The following simple recurrence formula will be very useful to work with the mortality table;

$$ {}_n p_x = \prod_{k=0}^{n-1} \left(1 - q_{x+k}\right), \quad n \geq 1. $$

As an exercise towards understanding the use of the table confirm the following.

i. The probability that Jill aged 60 survives beyond the age of 65 is 0.945398568
ii. The probability that Jane aged 62 survives beyond the age of 65 but passes away before the age of 68 is 0.04556584
iii. The probability that Julie aged 70 dying in the next two years is 0.045804275

From the mortality table one can determine the values of ${}_n\mathrm{p}_x$, ${}_n\mathrm{q}_x$ and ${}_n\mathrm{q}_x m$ for non-negative integral values for x, m and n, but not for non-negative reals in general. In other words, we can determine the distributions of $\{K(x)\}_{x\geq 0}$ but not of $\{T(x)\}_{x\geq 0}$. The short fall in the information is filled by the distributional assumption on the fractional age at death. One such assumption is the **uniform fractional age assumption** which is the assumption that

$$T(x) = K(x) + U, \quad \text{where } U \sim U(0,1) \text{ and } U \text{ is independent of } K(x).$$

As we have observed before, ${}_t\mathrm{p}_x = S_{T(x)}(t)$. This implies that specification of ${}_t\mathrm{p}_x$ for all non-negative values of t fixes the distribution of $T(x)$. But as

$${}_t\mathrm{p}_x = \left({}_{\lfloor t\rfloor}\mathrm{p}_x\right)_{t-\lfloor t\rfloor}\mathrm{p}_{x+\lfloor t\rfloor}, \quad \forall t \geq 0,$$

it suffices to specify just ${}_u\mathrm{q}_x$ for all values of u in the unit interval ($[0,1]$). An example of the above relationship is ${}_{2.5}\mathrm{p}_{20} = ({}_2\mathrm{p}_{20})_{0.5}\mathrm{p}_{22}$. Note that we have tacitly assumed that x is a non-negative integer. This need not be true and in this case we handle it as

$${}_{2.25}\mathrm{p}_{20.25} = ({}_{0.75}\mathrm{p}_{20.25})\mathrm{p}_{21}({}_{0.5}\mathrm{p}_{22}) = \left(\frac{\mathrm{p}_{20}}{{}_{0.25}\mathrm{p}_{20}}\right)\mathrm{p}_{21}({}_{0.5}\mathrm{p}_{22}).$$

Try generalizing the above as an exercise. Hence we shall in the below assume that x is a non-negative integer.

The following table summarizes the key relationships flowing out of the uniform assumption.

${}_u\mathrm{p}_x =$	${}_u\mathrm{q}_x =$	$\mu(x+u) =$	${}_v\mathrm{q}_{x+u} =$	${}_u\mathrm{p}_x\mu(x+u) =$
$1 - u(\mathrm{q}_x)$	$u(\mathrm{q}_x)$	$\frac{\mathrm{q}_x}{1-u(\mathrm{q}_x)}$	$\frac{v\mathrm{q}_x}{1-u(\mathrm{q}_x)}$, $v \leq 1-u$	q_x

Time to practice! Under the uniform fractional age assumption, using the mortality table confirm the following.

i. The probability that Jill aged 60.25 survives beyond the age of 65.5 is 0.940729062
ii. The probability that Jane aged 62.25 survives beyond the age of 65.75 but passes away before the age of 68.25 is 0.039449914
iii. The probability that Julie aged 70.25 dying in the next two and a quarter years is 0.053391516

Since the SOA exam deals with only the uniform fractional age assumption we shall restrict our attention to the same.

5 Moments and Recurrence Relations

We start by tabulating some general results on moments for non-negative continuous random variables and non-negative integer valued discrete type random variables.

	Continuous	**Non-Negative Integer Valued**
$\mathbb{E}(X) =$	$\int_0^\infty S(x)\mathrm{d}x$	$\sum_{k=1}^\infty \Pr(X \geq k)$
$\mathbb{E}\left(X^2\right) =$	$\int_0^\infty 2xS(x)\mathrm{d}x$	$\sum_{k=1}^\infty (2k-1)\Pr(X \geq k)$
$\mathbb{E}\left(X^j\right) =$	$\int_0^\infty jx^{j-1}S(x)\mathrm{d}x$	$\sum_{k=0}^\infty \Delta k^j \Pr(X \geq k+1)$

As an application of the above, we can write

$$\mathring{e}_x = \mathbb{E}(T(x)) = \int_0^\infty {}_t p_x \mathrm{d}t \quad \text{and} \quad e_x = \mathbb{E}(K(x)) = \sum_1^\infty {}_k p_x$$

and their generalization,

$$\mathring{e}_{x:\overline{n}|} = \mathbb{E}(T(x) \wedge n) = \int_0^n {}_t p_x \mathrm{d}t \quad \text{and} \quad e_{x:\overline{n}|} = \mathbb{E}(K(x) \wedge n) = \sum_1^n {}_k p_x.$$

An interesting way to look at $e_{x:\overline{n}|} = \mathbb{E}(K(x) \wedge n) = \sum_1^n {}_k p_x$ is through the relation

$$K(x) \wedge n = \sum_1^n I_{\{(x) \text{ is alive at age } x+j\}}.$$

Using the uniform fractional age assumption it is easy to conclude that

$$\mathring{e}_{x:\overline{n}|} - e_{x:\overline{n}|} = \frac{{}_n q_x}{2} \quad \text{which implies} \quad \mathring{e}_x - e_x = 0.5$$

and

$$\mathrm{Var}(T(x)) = \mathrm{Var}(K(x)) + \frac{1}{12} \quad \left(\tfrac{1}{12} \text{ being the variance of a } U(0,1)\right).$$

Now we come to recurrence relations - A useful tool for computation. First we note an important consequence of the uniform fractional age assumption.

> Under the uniform fractional age assumption
>
> $$T(x)|T(x) < 1 \stackrel{d}{=} U(0,1)$$

Now we can write a general recurrence formula from which most if not all recurrence formulae in Life contingencies will follow. Let $\psi(\cdot)$ be an arbitrary function. Then

$$\begin{aligned} E(\psi(T(x))) &= \mathbb{E}(\psi(T(x))\,|T(x) < 1)\Pr(T(x) < 1) + \mathbb{E}(\psi(T(x))\,|T(x) \geq 1)\Pr(T(x) \geq 1) \\ &= \mathrm{q}_x\mathbb{E}(\psi(U)) + \mathrm{p}_x\mathbb{E}(\psi(T(x+1)+1)), \qquad \text{(ALoM and Uniform Assumption)} \end{aligned}$$

The table below shows some applications:

$\psi(x) =$	**Recurrence Relation**
x	$\mathring{e}_x = p_x(\mathring{e}_{x+1} + 1) + \frac{q_x}{2}$
$x \wedge n$	$\mathring{e}_{x:\overline{n}\vert} = p_x(\mathring{e}_{x+1:\overline{n-1}\vert} + 1) + \frac{q_x}{2},\ n \geq 1$
$\lfloor x \rfloor$	$e_x = p_x(e_{x+1} + 1)$
$\lfloor x \rfloor \wedge n$	$e_{x:\overline{n}\vert} = p_x(e_{x+1:\overline{n-1}\vert} + 1),\ n \geq 1$

A few exercises for practice:

i. If $\mathring{e}_{25} = 52.34128493$ show $\mathring{e}_{26} = 51.40149066$ and $e_{24} = 52.78104586$.

ii. If $e_{40:\overline{5}\vert} = 4.959384381$ show $\mathring{e}_{40:\overline{5}\vert} = 4.966513497$ and $e_{42:\overline{3}\vert} = 2.981927494$.

6 Select & Ultimate Table

The way that a significant amount of life insurance is sold involves some kind of selection process. Depending on the stringency of the selection mechanism, the mortality rate of this selected population may be markedly/marginally lesser than the insured population at large, even accounting for age distribution. Hence it is now becoming increasingly common to use select and ultimate mortality rates and the aggregate table is slowly becoming extinct. Included in this note is the 1990-95 US SOA Basic Female, Age nearest select and ultimate table.

Notation: A life aged $x + n$ selected at age x is denoted by $[x] + n$. A newly selected life at age x is denoted by $[x]$.

Every select and ultimate table has a number n, called the period which is the smallest number k such that $q_{[x]+k} = q_{x+k}$, or the number of years after which the selection effect is deemed to be insignificant. Note that we do not employ a new notation for the ultimate rate to distinguish it from the aggregate rate - the context should make it clear. The 90-95 SOA Basic table has a period of 25 years.

It is instructive to know the population that the mortality rates pertain to. Let us consider an insured population with all having an age x. The mortality for this population is called the aggregate rate q_x. For estimating the select and ultimate rates with a period of 25, this population is broken down into 26 subgroups - first subgroup consisting of insureds who purchased and insurance policy the current year, second of those who last purchased an insurance policy a year back, ... the penultimate being of those who last purchased an insurance policy 24 years back and the ultimate subgroup consisting of those who last bought an insurance policy at least 25 years back. The k subgroup mortality is denoted by $q_{[x+1-k]+k-1}$, $k \leq 25$ and the 26^{th} subgroup mortality is the ultimate mortality. Hence it is easy to see that the aggregate mortality rate will be a weighted average of the select and ultimate mortality rates - the weights are difficult to discern from the tables but one can check that the aggregate mortality rate will always lie somewhere between the select and ultimate rates. Check for age 30 using the aggregate rate from the US CSO table!

The algorithm to work with a select and ultimate table is similar to that of the aggregate table, once the rates corresponding to the age at selection are chosen. In the table, the rates for a life selected at age 25 at various ages is highlighted in blue. To confirm your understanding, check that ${}_5p_{[25]+23} = 0.987472848$.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Ultimate	
0	0.0005	0.00034	0.00023	0.00016	0.00012	0.00011	0.00013	0.00015	0.00015	0.00016	0.00016	0.00017	0.00021	0.00024	0.00027	0.00028	0.00032	0.00033	0.00034	0.00036	0.00038	0.00039	0.00039	0.0004	0.00041	0.00041	25
1	0.00026	0.00021	0.00014	0.00011	0.0001	0.00013	0.00014	0.00015	0.00015	0.00016	0.00017	0.00019	0.00021	0.00025	0.00028	0.00032	0.00033	0.00034	0.00036	0.00038	0.00039	0.00039	0.0004	0.00041	0.00041	0.00042	26
2	0.00019	0.00013	0.0001	0.0001	0.00013	0.00014	0.00014	0.00015	0.00016	0.00017	0.00018	0.0002	0.00024	0.00028	0.00032	0.00033	0.00034	0.00036	0.00038	0.00039	0.00039	0.0004	0.00041	0.00041	0.00042	0.00044	27
3	0.00011	0.0001	0.00009	0.00012	0.00013	0.00014	0.00015	0.00016	0.00016	0.00017	0.00019	0.00024	0.00028	0.00032	0.00033	0.00034	0.00036	0.00038	0.00039	0.00039	0.0004	0.00041	0.00041	0.00042	0.00044	0.00047	28
4	0.00009	0.00009	0.00011	0.00013	0.00014	0.00014	0.00015	0.00016	0.00017	0.00019	0.00024	0.00028	0.00032	0.00033	0.00034	0.00036	0.00038	0.00039	0.00039	0.0004	0.00041	0.00041	0.00042	0.00044	0.00047	0.00048	29
5	0.00009	0.0001	0.00012	0.00013	0.00014	0.00015	0.00016	0.00016	0.00018	0.00023	0.00028	0.00032	0.00033	0.00034	0.00036	0.00038	0.00039	0.00039	0.0004	0.00041	0.00041	0.00042	0.00044	0.00047	0.00048	0.00051	30
6	0.0001	0.00011	0.00012	0.00013	0.00014	0.00015	0.00016	0.00018	0.00023	0.00028	0.00032	0.00033	0.00034	0.00034	0.00036	0.00039	0.00039	0.0004	0.00041	0.00041	0.00042	0.00044	0.00047	0.00048	0.00051	0.00056	31
7	0.00011	0.00011	0.00012	0.00014	0.00015	0.00016	0.00018	0.00023	0.00028	0.00032	0.00033	0.00033	0.00032	0.00036	0.00037	0.00039	0.0004	0.00041	0.00041	0.00042	0.00044	0.00047	0.00048	0.00051	0.00056	0.00063	32
8	0.00011	0.00011	0.00013	0.00015	0.00015	0.00018	0.00022	0.00028	0.00032	0.00033	0.00033	0.00032	0.00034	0.00036	0.00035	0.0004	0.00041	0.00041	0.00042	0.00044	0.00047	0.00048	0.00051	0.00056	0.00063	0.0007	33
9	0.00011	0.00012	0.00014	0.00015	0.00017	0.00022	0.00028	0.00032	0.00033	0.00033	0.00031	0.00034	0.00035	0.00035	0.00038	0.00041	0.00041	0.00042	0.00044	0.00047	0.00048	0.00051	0.00056	0.00063	0.0007	0.00077	34
10	0.00012	0.00013	0.00015	0.00017	0.00021	0.00028	0.00032	0.00033	0.00032	0.0003	0.00033	0.00034	0.00033	0.00037	0.00038	0.00041	0.00042	0.00044	0.00047	0.00048	0.00051	0.00056	0.00063	0.0007	0.00077	0.00083	35
11	0.00013	0.00014	0.00017	0.00021	0.00028	0.00032	0.00033	0.00032	0.0003	0.00032	0.00033	0.00033	0.00036	0.00037	0.0004	0.00042	0.00044	0.00047	0.00048	0.00051	0.00056	0.00063	0.0007	0.00077	0.00083	0.00088	36
12	0.00014	0.00016	0.0002	0.00026	0.00032	0.00033	0.00031	0.0003	0.00031	0.00032	0.00032	0.00036	0.00037	0.0004	0.00039	0.00044	0.00047	0.00048	0.00051	0.00056	0.00063	0.0007	0.00077	0.00083	0.00088	0.00093	37
13	0.00016	0.00018	0.00024	0.0003	0.00032	0.00031	0.00029	0.00031	0.00031	0.00032	0.00035	0.00036	0.00037	0.00039	0.00042	0.00047	0.00048	0.00051	0.00056	0.00063	0.0007	0.00077	0.00083	0.00088	0.00093	0.00096	38
14	0.00018	0.00023	0.00028	0.00031	0.0003	0.00029	0.0003	0.00031	0.00031	0.00034	0.00035	0.00035	0.00037	0.0004	0.00044	0.00048	0.00051	0.00056	0.00063	0.0007	0.00077	0.00083	0.00088	0.00093	0.00096	0.00096	39
15	0.00018	0.00028	0.00031	0.0003	0.00028	0.0003	0.0003	0.0003	0.00032	0.00034	0.00034	0.00036	0.00039	0.00043	0.00045	0.00051	0.00056	0.00063	0.0007	0.00077	0.00083	0.00088	0.00093	0.00096	0.00096	0.00096	40
16	0.00028	0.0003	0.00029	0.00028	0.0003	0.0003	0.00029	0.00032	0.00033	0.00034	0.00035	0.00037	0.0004	0.00043	0.00047	0.00056	0.00063	0.0007	0.00077	0.00083	0.00088	0.00093	0.00096	0.00096	0.00096	0.001	41
17	0.0003	0.00029	0.00027	0.00029	0.0003	0.00029	0.00029	0.00033	0.00033	0.00034	0.00036	0.00039	0.00042	0.00046	0.00053	0.00063	0.0007	0.00077	0.00083	0.00088	0.00093	0.00096	0.00096	0.00096	0.001	0.00107	42
18	0.00027	0.00025	0.00028	0.0003	0.00029	0.00028	0.0003	0.00032	0.00033	0.00035	0.00038	0.0004	0.00045	0.00053	0.00061	0.0007	0.00077	0.00083	0.00088	0.00093	0.00096	0.00096	0.00096	0.001	0.00107	0.00119	43
19	0.00023	0.00027	0.0003	0.00028	0.00028	0.00029	0.00031	0.00032	0.00034	0.00037	0.00039	0.00044	0.00053	0.00061	0.00069	0.00077	0.00083	0.00088	0.00093	0.00096	0.00096	0.00096	0.001	0.00107	0.00119	0.00132	44
20	0.00022	0.00026	0.00027	0.00028	0.00029	0.0003	0.0003	0.00033	0.00035	0.00038	0.00043	0.0005	0.00057	0.00066	0.00074	0.00081	0.00088	0.00093	0.00096	0.00096	0.00096	0.001	0.00107	0.00119	0.00132	0.00145	45
21	0.00019	0.00022	0.00024	0.00027	0.00028	0.00029	0.00032	0.00035	0.00037	0.00041	0.00047	0.00054	0.00061	0.0007	0.00079	0.00086	0.00092	0.00096	0.00096	0.00096	0.001	0.00107	0.00119	0.00132	0.00145	0.00162	46
22	0.00016	0.00019	0.00022	0.00026	0.00029	0.00031	0.00033	0.00036	0.0004	0.00046	0.00053	0.0006	0.00069	0.00074	0.00083	0.0009	0.00095	0.00095	0.00096	0.001	0.00107	0.00119	0.00132	0.00145	0.00162	0.0018	47
23	0.00016	0.00019	0.00022	0.00025	0.00029	0.00032	0.00034	0.00038	0.00045	0.00053	0.0006	0.00067	0.00073	0.00079	0.00088	0.00094	0.00095	0.00096	0.001	0.00107	0.00119	0.00132	0.00145	0.00162	0.0018	0.002	48
24	0.00016	0.00019	0.00022	0.00026	0.0003	0.00032	0.00036	0.00042	0.00052	0.00059	0.00067	0.00073	0.00079	0.00084	0.00092	0.00094	0.00096	0.00099	0.00107	0.00119	0.00132	0.00145	0.00162	0.0018	0.002	0.00222	49

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Ultimate	
25	0.00016	0.00019	0.00023	0.00027	0.00029	0.00033	0.00039	0.00048	0.00059	0.00066	0.00072	0.00079	0.00083	0.00088	0.00093	0.00095	0.00099	0.00107	0.00119	0.00132	0.00145	0.00162	0.0018	0.002	0.00222	0.00247	50
26	0.00016	0.0002	0.00024	0.00027	0.00029	0.00035	0.00043	0.00054	0.00065	0.00072	0.00079	0.00081	0.00081	0.00089	0.00095	0.00099	0.00107	0.00119	0.00132	0.00145	0.00162	0.0018	0.002	0.00222	0.00247	0.00277	51
27	0.00017	0.0002	0.00024	0.00027	0.00031	0.00038	0.00047	0.00059	0.00072	0.00078	0.0008	0.00079	0.00082	0.00091	0.00099	0.00107	0.00119	0.00132	0.00145	0.00162	0.0018	0.002	0.00222	0.00247	0.00277	0.00313	52
28	0.00017	0.0002	0.00025	0.0003	0.00035	0.00041	0.0005	0.00061	0.00072	0.00078	0.00078	0.00078	0.00083	0.00095	0.00107	0.00119	0.00132	0.00145	0.00162	0.0018	0.002	0.00222	0.00247	0.00277	0.00313	0.00352	53
29	0.00017	0.00021	0.00027	0.00033	0.00038	0.00045	0.00053	0.00062	0.00072	0.00077	0.00076	0.00078	0.00087	0.00104	0.00119	0.00132	0.00145	0.00162	0.0018	0.00199	0.00222	0.00247	0.00277	0.00313	0.00352	0.00393	54
30	0.00017	0.00022	0.00029	0.00036	0.00042	0.00048	0.00054	0.00063	0.0007	0.00073	0.00075	0.00081	0.00094	0.00116	0.00132	0.00145	0.00162	0.0018	0.00199	0.00221	0.00247	0.00277	0.00313	0.00352	0.00393	0.00437	55
31	0.00018	0.00024	0.00031	0.00038	0.00045	0.0005	0.00056	0.00062	0.00066	0.0007	0.00076	0.00088	0.00105	0.00129	0.00145	0.00162	0.00179	0.00199	0.00221	0.00246	0.00277	0.00312	0.00352	0.00393	0.00437	0.00485	56
32	0.00019	0.00026	0.00033	0.0004	0.00047	0.00052	0.00059	0.0006	0.00067	0.00073	0.0008	0.00097	0.00118	0.00143	0.00162	0.00179	0.00199	0.00221	0.00246	0.00277	0.00311	0.0035	0.00393	0.00437	0.00485	0.00536	57
33	0.0002	0.00027	0.00035	0.00043	0.00051	0.00056	0.00059	0.00064	0.0007	0.0008	0.00095	0.00113	0.00133	0.0016	0.00179	0.00199	0.00221	0.00246	0.00276	0.0031	0.00348	0.00389	0.00437	0.00485	0.00536	0.00592	58
34	0.00021	0.00028	0.00037	0.00046	0.00054	0.00059	0.00063	0.0007	0.0008	0.00094	0.00112	0.00132	0.00158	0.00178	0.00199	0.00221	0.00246	0.00276	0.00307	0.00345	0.00385	0.0043	0.00485	0.00536	0.00592	0.00652	59
35	0.00021	0.00029	0.00038	0.00048	0.00056	0.00062	0.00068	0.00079	0.00094	0.00111	0.00131	0.00155	0.00172	0.00198	0.0022	0.00245	0.00275	0.00307	0.00341	0.00381	0.00426	0.00475	0.00531	0.00592	0.00652	0.00709	60
36	0.0002	0.00029	0.00039	0.00049	0.00058	0.00067	0.00077	0.00092	0.00111	0.0013	0.00153	0.00169	0.00195	0.0022	0.00244	0.00275	0.00305	0.00338	0.00377	0.00423	0.00471	0.00525	0.00585	0.00652	0.00709	0.00771	61
37	0.0002	0.00029	0.00039	0.0005	0.00061	0.00075	0.0009	0.00107	0.00128	0.00152	0.00167	0.00194	0.00217	0.00242	0.00272	0.00304	0.00336	0.00373	0.00419	0.00471	0.00523	0.0058	0.00643	0.00706	0.00771	0.00838	62
38	0.00022	0.00032	0.00042	0.00053	0.00067	0.00083	0.00099	0.00116	0.0014	0.00165	0.00193	0.00215	0.00239	0.00268	0.00297	0.00334	0.00367	0.00417	0.00468	0.0052	0.00574	0.00633	0.00704	0.00769	0.00834	0.00907	63
39	0.00024	0.00035	0.00046	0.00059	0.00076	0.00093	0.00109	0.00128	0.00152	0.00178	0.00206	0.00231	0.00261	0.00293	0.00323	0.00365	0.0041	0.00467	0.00517	0.00568	0.00626	0.00695	0.00767	0.0083	0.00898	0.00981	64
40	0.00026	0.00038	0.00052	0.00068	0.00085	0.00102	0.00121	0.0014	0.00165	0.00191	0.00221	0.0025	0.00285	0.00318	0.00349	0.00399	0.00458	0.00515	0.00562	0.0062	0.00688	0.00756	0.00826	0.00889	0.00971	0.01074	65
41	0.00029	0.00044	0.00061	0.00078	0.00095	0.00114	0.00134	0.00154	0.00179	0.00206	0.00237	0.00272	0.00309	0.00344	0.00376	0.00445	0.00508	0.00556	0.00616	0.00681	0.00748	0.00821	0.00885	0.00967	0.01067	0.0119	66
42	0.00033	0.00052	0.0007	0.00088	0.00107	0.00127	0.00148	0.00169	0.00194	0.00223	0.00256	0.00293	0.00333	0.00371	0.00415	0.00493	0.0055	0.0061	0.00674	0.00737	0.00817	0.0088	0.00962	0.01064	0.01184	0.01314	67
43	0.00037	0.00059	0.00079	0.00101	0.00122	0.00143	0.00166	0.00188	0.00216	0.00249	0.00285	0.00326	0.00371	0.00414	0.00456	0.00542	0.00603	0.00667	0.00725	0.00805	0.00871	0.00957	0.01058	0.01181	0.013	0.01427	68
44	0.00041	0.00066	0.00091	0.00115	0.00139	0.00162	0.00186	0.0021	0.00241	0.00276	0.00316	0.00362	0.00414	0.00455	0.00503	0.00596	0.0066	0.00721	0.00796	0.00866	0.00947	0.01053	0.0118	0.01297	0.01406	0.0155	69
45	0.00045	0.00074	0.00103	0.00131	0.00158	0.00183	0.00209	0.00239	0.00268	0.00305	0.0035	0.00404	0.00453	0.00501	0.00554	0.00647	0.0071	0.00788	0.00862	0.00942	0.01048	0.01178	0.01287	0.01392	0.01519	0.01686	70
46	0.0005	0.00084	0.00117	0.00149	0.0018	0.00208	0.00237	0.00267	0.00297	0.00337	0.00388	0.00447	0.00498	0.00553	0.00607	0.00701	0.0078	0.00857	0.00937	0.01042	0.01176	0.01279	0.01385	0.01504	0.01644	0.01842	71
47	0.00055	0.00094	0.00133	0.0017	0.00205	0.00237	0.00266	0.00294	0.00329	0.00372	0.00428	0.00495	0.00546	0.00601	0.00669	0.00771	0.00853	0.00932	0.01037	0.01173	0.01274	0.0137	0.01488	0.01635	0.01787	0.02031	72
48	0.00061	0.00105	0.00148	0.00187	0.00223	0.00255	0.00288	0.0032	0.0036	0.00406	0.00466	0.00539	0.006	0.00664	0.00737	0.00844	0.00927	0.01031	0.01169	0.01268	0.01356	0.01473	0.01618	0.01769	0.0197	0.02262	73
49	0.00066	0.00116	0.00166	0.00206	0.00241	0.00274	0.00311	0.00349	0.00392	0.00442	0.00506	0.0058	0.00663	0.00733	0.0081	0.00922	0.01026	0.01166	0.01257	0.01349	0.01457	0.01601	0.0175	0.01949	0.02194	0.02511	74

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Ultimate	
50	0.00073	0.0013	0.00187	0.00226	0.00259	0.00293	0.00336	0.00379	0.00426	0.0048	0.00544	0.00625	0.00719	0.00807	0.00889	0.01021	0.0116	0.01248	0.01342	0.01442	0.01568	0.01732	0.01929	0.02183	0.02448	0.02734	75
51	0.0008	0.00146	0.00209	0.00247	0.00277	0.00314	0.00362	0.0041	0.00463	0.00516	0.00584	0.00672	0.00777	0.00887	0.00991	0.01154	0.01241	0.01327	0.01426	0.01544	0.01695	0.01909	0.02172	0.02436	0.0268	0.02924	76
52	0.00088	0.00163	0.00232	0.00269	0.00297	0.00335	0.00388	0.00443	0.00496	0.00554	0.00627	0.00721	0.0084	0.00986	0.01126	0.01235	0.01313	0.01411	0.015	0.01658	0.01888	0.0216	0.02423	0.02666	0.02872	0.03152	77
53	0.00098	0.00175	0.00246	0.00286	0.00316	0.00356	0.00411	0.00465	0.00523	0.00585	0.00659	0.00758	0.00894	0.01057	0.01194	0.01299	0.01395	0.01483	0.0164	0.01868	0.02138	0.0241	0.02652	0.02866	0.03098	0.03459	78
54	0.00108	0.00188	0.0026	0.00301	0.00336	0.00377	0.00429	0.00487	0.00551	0.00615	0.00691	0.00804	0.00959	0.01124	0.0124	0.01364	0.0145	0.01603	0.01828	0.02104	0.02385	0.02639	0.02851	0.03095	0.0339	0.03851	79
55	0.00119	0.00202	0.00273	0.00316	0.00355	0.00395	0.00447	0.0051	0.00578	0.00646	0.00732	0.0086	0.01019	0.01172	0.01285	0.01416	0.01566	0.01787	0.02059	0.0236	0.02611	0.02837	0.03073	0.03387	0.03755	0.04293	80
56	0.00131	0.00215	0.00285	0.00331	0.00371	0.00413	0.00465	0.00531	0.00605	0.00685	0.00781	0.00911	0.01063	0.01219	0.01335	0.01529	0.01746	0.02036	0.02335	0.02584	0.02822	0.03048	0.03373	0.03736	0.04164	0.04818	81
57	0.00144	0.00229	0.00297	0.00341	0.00387	0.00431	0.00481	0.00552	0.00639	0.0076	0.00882	0.00993	0.01105	0.01271	0.01423	0.01706	0.01991	0.0231	0.0257	0.02807	0.03026	0.03356	0.03728	0.04143	0.04654	0.05405	82
58	0.00147	0.00244	0.00324	0.00382	0.0043	0.00478	0.00541	0.00632	0.00739	0.00856	0.00965	0.011	0.01212	0.0141	0.01597	0.01946	0.02285	0.02543	0.02793	0.03016	0.03338	0.03705	0.04121	0.04639	0.05113	0.0599	83
59	0.0015	0.00257	0.00352	0.00427	0.00475	0.00529	0.00615	0.00729	0.00846	0.00952	0.01086	0.01149	0.01341	0.01585	0.01849	0.02244	0.02516	0.02778	0.03004	0.03321	0.0369	0.04113	0.04625	0.05097	0.05553	0.06635	84
60	0.00149	0.0027	0.00384	0.00475	0.00529	0.00593	0.00703	0.00833	0.00949	0.01071	0.01147	0.01279	0.01502	0.01806	0.02134	0.02488	0.02764	0.02994	0.033	0.03674	0.04095	0.04615	0.05081	0.05541	0.06276	0.07352	85
61	0.00147	0.00283	0.00417	0.00528	0.00587	0.00669	0.00798	0.00933	0.01063	0.01146	0.01274	0.01496	0.01706	0.02051	0.02413	0.02749	0.02979	0.0328	0.03659	0.04087	0.04606	0.05076	0.05529	0.06263	0.06911	0.08263	86
62	0.00144	0.00296	0.00453	0.00567	0.00662	0.0075	0.00888	0.01043	0.01146	0.01265	0.01476	0.01689	0.01931	0.02283	0.02672	0.02963	0.03266	0.03647	0.04078	0.04596	0.05065	0.05517	0.0625	0.06874	0.0785	0.09283	87
63	0.0016	0.0031	0.0047	0.00617	0.00683	0.00761	0.00899	0.01064	0.0125	0.01471	0.01672	0.01914	0.02161	0.02502	0.02897	0.03252	0.03632	0.04065	0.04582	0.05054	0.05511	0.06237	0.06837	0.07767	0.08865	0.1035	88
64	0.00179	0.00327	0.0049	0.00633	0.0069	0.00769	0.00911	0.01092	0.01325	0.01624	0.01896	0.02108	0.02371	0.02756	0.03193	0.0362	0.04057	0.04572	0.05043	0.05499	0.06203	0.068	0.07684	0.08772	0.09936	0.11482	89
65	0.00201	0.00348	0.00505	0.00634	0.00693	0.00777	0.00926	0.0113	0.01421	0.01787	0.02108	0.02314	0.02615	0.03085	0.03565	0.04044	0.04562	0.05032	0.05487	0.0617	0.06764	0.07602	0.08679	0.09832	0.1108	0.1249	90
66	0.00228	0.00367	0.00509	0.00632	0.00694	0.00787	0.00948	0.0118	0.01517	0.01928	0.02298	0.02554	0.0293	0.03497	0.03983	0.04553	0.05022	0.05475	0.06137	0.06727	0.07519	0.08605	0.09729	0.11022	0.12115	0.13378	91
67	0.00257	0.00378	0.0051	0.00627	0.00747	0.0088	0.00978	0.01222	0.01585	0.0204	0.02518	0.02864	0.03325	0.03966	0.04478	0.05011	0.05463	0.06104	0.0669	0.07478	0.0854	0.09625	0.10908	0.1199	0.12977	0.14397	92
68	0.00289	0.00424	0.0057	0.00706	0.00811	0.00966	0.01173	0.01406	0.01736	0.02174	0.02656	0.03062	0.03607	0.04354	0.04928	0.05451	0.06071	0.06653	0.07437	0.08494	0.09574	0.10793	0.11928	0.1291	0.13894	0.15942	93
69	0.00324	0.00476	0.00642	0.00804	0.00954	0.01152	0.01373	0.01584	0.01912	0.02354	0.02833	0.0327	0.03936	0.04775	0.05359	0.05971	0.06617	0.07354	0.08429	0.09522	0.10735	0.11865	0.12843	0.13894	0.15463	0.17917	94
70	0.00363	0.00538	0.00731	0.00928	0.01116	0.01343	0.0157	0.01855	0.02139	0.02583	0.03018	0.03509	0.04289	0.05175	0.05829	0.06543	0.07271	0.08355	0.09522	0.10678	0.11865	0.12776	0.13822	0.15384	0.17379	0.20416	95
71	0.00411	0.00616	0.00843	0.01068	0.01278	0.01529	0.01798	0.02113	0.02424	0.02835	0.03229	0.03755	0.04616	0.05608	0.06344	0.07189	0.08262	0.09418	0.1062	0.11803	0.12709	0.1375	0.15344	0.1729	0.19906	0.23294	96
72	0.00471	0.00713	0.0097	0.01203	0.01432	0.01746	0.02088	0.02381	0.02748	0.0313	0.03445	0.03963	0.04966	0.06081	0.07001	0.08169	0.09315	0.10563	0.11741	0.12642	0.13678	0.15304	0.1729	0.19803	0.22828	0.25777	97
73	0.00566	0.00753	0.01094	0.01316	0.01576	0.01903	0.02311	0.02619	0.03035	0.03459	0.03894	0.04445	0.05514	0.0661	0.0789	0.09004	0.10333	0.11616	0.12575	0.13678	0.15304	0.1729	0.19803	0.22828	0.25777	0.25991	98
74	0.00628	0.0082	0.0117	0.01418	0.0173	0.02118	0.02576	0.02939	0.03405	0.03894	0.04313	0.04926	0.06197	0.07426	0.08797	0.10104	0.11491	0.12575	0.13678	0.15304	0.17245	0.19803	0.22816	0.25777	0.25991	0.27242	99

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	Ultimate	
75	0.00684	0.00877	0.01261	0.01557	0.01926	0.02361	0.02891	0.03297	0.03774	0.04313	0.04779	0.05536	0.06962	0.08487	0.09989	0.11241	0.12308	0.13534	0.15304	0.172	0.19803	0.22805	0.25777	0.25991	0.27242	0.29421	100
76	0.00731	0.00946	0.01384	0.01733	0.02146	0.0265	0.03243	0.03654	0.0418	0.04779	0.05371	0.06219	0.07866	0.09415	0.10991	0.1204	0.13534	0.15304	0.172	0.19701	0.22781	0.25777	0.25991	0.27242	0.29421	0.31922	101
77	0.00788	0.01038	0.01541	0.01932	0.02409	0.02973	0.03594	0.04047	0.04632	0.05371	0.06034	0.07245	0.08956	0.10367	0.11773	0.13246	0.15145	0.172	0.19701	0.22758	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	102
78	0.00865	0.01155	0.01717	0.02168	0.02703	0.03295	0.03981	0.04485	0.05206	0.06034	0.07038	0.08382	0.09992	0.11505	0.12958	0.14826	0.17021	0.19701	0.22735	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	103
79	0.00963	0.01288	0.01927	0.02432	0.02995	0.03649	0.04411	0.0504	0.05848	0.07038	0.08267	0.09742	0.11371	0.1267	0.14507	0.16663	0.19599	0.22665	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	104
80	0.01073	0.01445	0.02162	0.02696	0.03317	0.04043	0.04958	0.05755	0.06831	0.08267	0.09492	0.10435	0.12382	0.14188	0.16483	0.19395	0.22595	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	105
81	0.01204	0.01622	0.02396	0.02986	0.03896	0.04792	0.0557	0.06727	0.08037	0.09367	0.10167	0.11518	0.13869	0.16125	0.18987	0.22374	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	106
82	0.01351	0.01797	0.02654	0.03455	0.04545	0.05384	0.0652	0.07807	0.08993	0.10034	0.11374	0.13243	0.15679	0.18746	0.22357	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	107
83	0.01498	0.02203	0.03161	0.04131	0.05106	0.06313	0.07578	0.08743	0.09785	0.11252	0.132	0.15643	0.18717	0.22341	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	108
84	0.01858	0.02819	0.03718	0.04834	0.05951	0.07463	0.08493	0.09729	0.11174	0.13155	0.15606	0.18689	0.22325	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	109
85	0.02426	0.03593	0.04529	0.05887	0.07144	0.08422	0.09699	0.11125	0.13087	0.15568	0.1866	0.22309	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	110
86	0.03188	0.04513	0.05581	0.07081	0.08362	0.09643	0.11099	0.13042	0.15511	0.18632	0.22292	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	111
87	0.04099	0.05563	0.06779	0.08301	0.09588	0.11049	0.1302	0.15473	0.18589	0.22276	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	112
88	0.05149	0.06762	0.08014	0.09532	0.10998	0.12975	0.15453	0.1856	0.2225	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	113
89	0.06353	0.07997	0.09268	0.10948	0.1293	0.15416	0.18544	0.22234	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	114
90	0.07608	0.09254	0.10712	0.12886	0.15378	0.18515	0.22227	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	115
91	0.08896	0.10699	0.12677	0.15342	0.18489	0.22211	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	116
92	0.10378	0.12666	0.15165	0.1846	0.22194	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	117
93	0.12382	0.15154	0.18325	0.22178	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	0.9874	118
94	0.14916	0.18317	0.22101	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	0.9874	0.99233	119
95	0.18135	0.22097	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	0.9874	0.99233	1	120
96	0.21994	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	0.9874	0.99233	1	0	121
97	0.25777	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	0.9874	0.99233	1	0	0	122
98	0.25991	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	0.9874	0.99233	1	0	0	0	123
99	0.27242	0.29421	0.31922	0.34795	0.38101	0.41911	0.46311	0.51406	0.57574	0.65059	0.74167	0.85292	0.91215	0.93952	0.95831	0.96789	0.97273	0.9776	0.98248	0.9874	0.99233	1	0	0	0	0	124