Problem 1

$$\theta = \frac{\Phi^{-1}(0.95)}{\sqrt{1000}} \cdot \frac{\sqrt{Var(X)}}{EX}$$

Hence $$\frac{\theta_A}{\theta_B} = \sqrt{\frac{Var(X_A)}{Var(X_B)}} \cdot \frac{EX_B}{EX_A}$$

$$EX_A = \frac{0.5+4.5}{2} = 2.5$$

$$Var(X_A) = \frac{5^2-1}{12} = 2$$ [Same as that for Disc. Unif. on $\{1,2,..5\}$ $DU(1,n) \rightarrow \frac{n^2-1}{12} = \frac{5^2-1}{12}$.]

$$EX_B = 5/2 = 2.5$$

$$Var(X_B) = \frac{5^2}{12}$$

Hence $$\frac{\theta_A}{\theta_B} = \sqrt{\frac{24}{25}} = 0.4\sqrt{6} = 0.979$$

Problem 2

Case of R_1

$$X \sim f(x) = \begin{cases} \dfrac{x}{0.16} & 0 < x < 0.4 \\ \dfrac{0.8-x}{0.16} & 0.4 < x < 0.8 \\ 0 & \text{otherwise} \end{cases}$$

$\boxed{\text{Symmetry about } 0.4 \Rightarrow EX = 0.4.}$

$$EX^2 = \int_0^{0.4} \frac{x^3}{0.16} + \int_{0.4}^{0.8} \frac{x^2(0.8-x)}{0.16}\,dx = 0.18\overline{6}$$

$$ER_1 = 0.36 * 0.8 + 0.48 * \left[\frac{0.4+0.8}{2}\right] + 0.16 * 0.4$$

$$= \underline{\underline{0.64}}$$

$$E\,\cancel{Rea}\,(R_1^2) = 0.36 * 0.8^2 + 0.48 * (0.4)^2\left(\frac{2^3-1}{3}\right) + 0.16 * 0.18\bar{6}$$

$$= 0.4394\bar{6}$$

$$\Rightarrow \quad Var(R_1) = \underline{\underline{0.029867}}$$

<u>Case of R_2</u>

$$X \sim f(x) = \begin{cases} \dfrac{x}{0.32} & 0 < x < 0.8 \\ 0 & \text{otherwise} \end{cases} \qquad EX = \frac{0.8^3}{0.32 * 3} = 0.5\bar{3}$$

$$EX^2 = \frac{(0.8)^4}{0.32 * 4} = 0.32$$

$$ER_2 = 0.68 * 0.8 + 0.32 * 0.5\bar{3} = \underline{\underline{0.714\bar{6}}}$$

$$ER_2^2 = 0.68 * 0.8^2 + 0.32 * 0.32 = 0.5376$$

$$\Rightarrow \quad Var(R_2) = \underline{\underline{0.02687}}$$

<u>Problem 3</u> (i) $\dfrac{825}{700} = 1 + \theta \qquad \theta = 17.8571\%.$

(ii) $RC \rightarrow$ Retained Claims $\qquad RP \rightarrow$ Retained Premiums.

$$\text{Prob. of loss} \approx 1 - \Phi\left(\frac{RP - E(RC)}{\sqrt{Var(RC)}}\right)$$

Current Situation

$E(RC) = 592.4$ (given)

$Var(RC) = 1369.00512$ (given)

$RP = 825 - 1.25\,(700 - 592.5) = 690.5$

Zero-Reinsurance

$E(RC) = 700$

$Var(RC) = 2587.2$

$RP = (1+\theta)700 - 0$

$= (1+\theta)700.$

The equation then is,

$$\frac{\overbrace{690.5}^{RP} - \overbrace{592.4}^{E(RC)}}{\sqrt{\underbrace{1369.00512}_{Var(RC)}}} = \frac{\overbrace{(1+\theta)700}^{RP} - \overbrace{700}^{E(RC)}}{\sqrt{\underbrace{2587.2}_{Var(RC)}}}$$

$\Rightarrow \quad \theta = 19.2656\,\%.$

(iii). Since the insurance company is charging more than the reinsurance premiums, with full reinsurance the insurance company has change left in its pocket with no claims to attend and hence the prob. of loss is zero, the ~~best~~ smallest it can ever be.