Method I

The probabilistic statements can be summarized as

$$\begin{tikzcd}[column sep=huge]
 & 0 \\
X \arrow[ur, "0.9"] \arrow[r, "0.05"] \arrow[dr, "0.05"'] & 250{,}000 \\
 & U(0,\ 250{,}000)
\end{tikzcd}$$

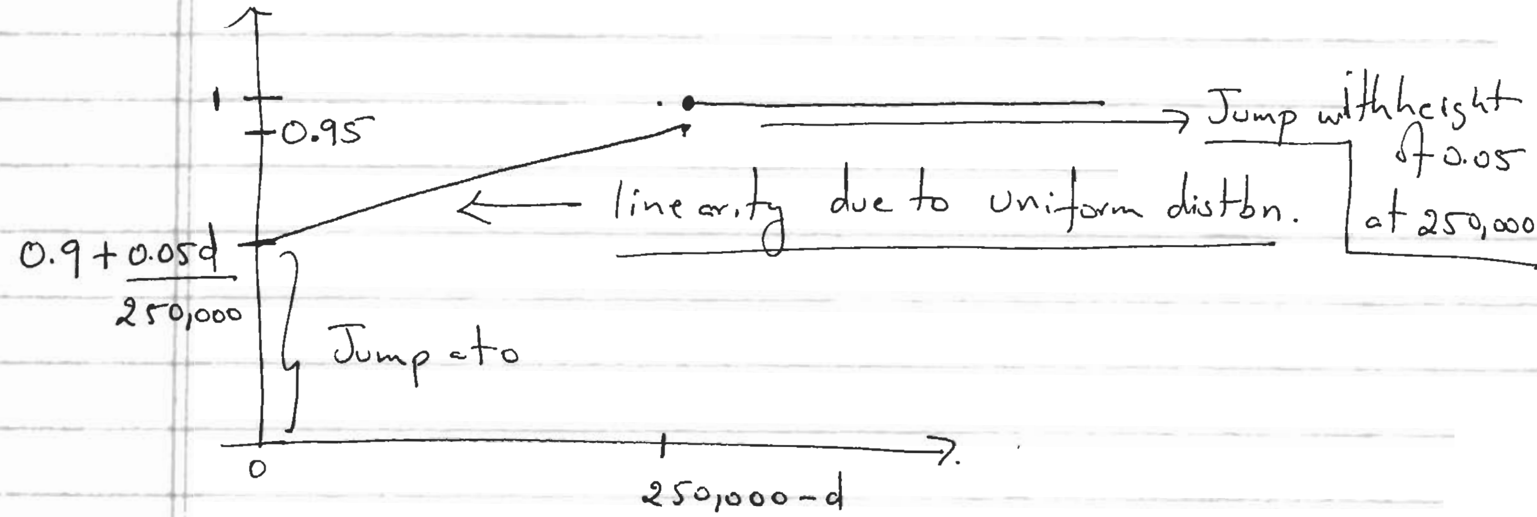
Let $d > 0$. Then

$$\begin{tikzcd}[column sep=huge]
 & 0 & \\
(X-d)_+ \arrow[ur, "0.9"] \arrow[r, "0.05"'] \arrow[dr, "0.05"'] & (250{,}000-d)_+ & \\
 & \bullet \arrow[r, "U<d"] \arrow[dr, "U>d"'] & 0 \\
 & U \sim U(0,250{,}000) & (U-d)
\end{tikzcd}$$

Hence, if $0 < d < 250{,}000$

$$\begin{tikzcd}[column sep=huge]
 & 0 \\
(X-d)_+ = \arrow[ur, "0.9+0.05\frac{d}{250{,}000}"] \arrow[r, "\underbrace{0.05\left(1-\frac{d}{250{,}000}\right)}_{0.05\Pr(U>d)}"'] \arrow[dr, "0.05"'] & U(0,\ 250{,}000-d) \\
 & (250{,}000-d)
\end{tikzcd}$$

As $U-d \mid U>d \sim U(0, 250{,}000-$



From the above one can conclude that the distbn. fn. is given by F.

<u>Method II</u> Let I be a random variable taking values 1, 2 and 3 with probabilities, 0.9, 0.05 and 0.05 respec

Then the probabilistic statements can be summarized as,

$$Pr(X=0 \mid I=1) = 1$$

$$Pr(X=250{,}000 \mid I=2) = 1$$

$$Pr(X \le x \mid I=3) = \begin{cases} \dfrac{x}{250{,}000} & 0 \le x < 250{,}000 \\ 0 & x < 0 \\ 1 & x \geq 250{,}000. \end{cases}$$

Now for $0 \le d \le 250{,}000$ we have

$$Pr\left((X-d)_+ = 0 \mid I=1 \right) = 1$$

$$Pr\left((X-d)_+ = 250{,}000-d \mid I=2 \right) = 1$$

~~and~~

$$Pr\left((X-d)_+ = 0 \mid I=3 \right) = Pr\left(X \le d \mid I=3 \right) = \frac{d}{250{,}000}$$

and

$$Pr\left((X-d)_+ \le y \mid I=3 \right) = \begin{cases} \dfrac{d+y}{250{,}000} & 0 \le y \le 250{,}000-d \\ 0 & y < 0. \\ 1 & y > 250{,}000-d \end{cases}$$

Now by Bayes theorem, one can conclude that $(X-d)_+ \sim F$.

$Pr(X=0) = 0.9$

$Pr(X=250,000) = 0.05$

$Pr(a < X \leq b) = \left[\frac{b-a}{250,000}\right] * 0.05$ for $0 \leq a < X < b \leq 250,000$

Hence, if $Y = (X-d)_+$, then

$Pr(Y=0) = Pr(X=0) + Pr(0 < X \leq d)$

$= 0.9 + \frac{d}{250,000} * 0.05$

$Pr(Y = 250,000 - d) = Pr(X = 250,000) = 0.05$

For $x > 0$ and $x < 250,000 - d$.

$Pr(Y \leq x) = Pr(Y=0) + Pr(0 < Y \leq x)$

$= Pr(Y=0) + Pr(d < X \leq x+d) = 0.9 + \frac{d}{250,000} * 0.05 + \frac{0.05\ ((x+d)-d)}{250,000}$

$= 0.9 + \frac{(x+d) * 0.05}{250,000}$

Hence $(X-d)_+$ has the distbn. fn. given in part (a).

(b) $E((X-d)_+) = 0.9 * 0 + 0.05 * (250{,}000 - d)$

$$+ 0.05 \int_0^{250{,}000} \frac{(x-d)_+}{250{,}000} dx$$

$$= 0.05 * (250{,}000 - d) + 0.05 \int_d^{250{,}000} \frac{(x-d)}{250{,}000} dx$$

$$= 0.05 \, (250{,}000 - d) + \frac{0.05}{250{,}000} \frac{(250{,}000-d)^2}{2}.$$

(c) $E[(X-d)_+] = 15{,}000$

$$0.05 \, (250{,}000 - d) + \frac{0.05}{250{,}000} \frac{(250{,}000-d)^2}{2} = 15{,}000$$

$$\Longleftrightarrow \left(1 - \frac{d}{250{,}000}\right) + 0.5\left(1 - \frac{d}{250{,}000}\right)^2 = \frac{15{,}000}{250{,}000} * \frac{1}{0.05}$$

Let $x = 1 - \frac{d}{250{,}000}$ then we have

$$x + 0.5x^2 = 1.2$$

$$\text{or } x = \frac{-1 \mp \sqrt{1 - 4*(-1.2)*0.5}}{2*0.5}$$

$$= -1 \mp \sqrt{3.4}$$

or $x = -2.8439$ or $0.8439 \Rightarrow \underline{d = 39{,}022.78}$

(d) The equation is as always

$$E\Big(U\big(w_0 - X + I(X) - \text{Premium}\big)\Big)$$

$$= E\big(U(w_0 - X)\big) \quad -(1)$$

Since the utility fn. is exponential, the equation is invariant with respect to w_0 and hence we may choose it to take any particular value. Let us take $w_0 = \text{Premium}$. Then (1) becomes

$$E\big(U(I(X) - X)\big) = E\big(U(\text{Premium} - X)\big).$$

In our case $\quad -X + I(X) = \begin{cases} -x & x \le d \\ -d & x \geq d \end{cases}$

Hence

$$E\big(U(I(X)-X)\big) = 0.9 + 0.05\, e^{d * 0.0001} + 0.05 \int_0^d \frac{e^{0.00001 * x}}{250{,}000}\, dx$$

$$+ 0.05 \int_d^{250{,}000} e^{0.00001 * d}\, dx \quad (2)$$

and

$$E\left(U(\text{Premium} - X)\right)$$

$$= e^{-0.00001 * \text{Premium}} \left[0.9 + 0.05\, e^{0.00001 * 250{,}000} + 0.05 \frac{\left[e^{0.00001 * 250{,}000} - 1 \right]}{250{,}000 \; 0.00001} \right] \quad (3)$$

Equating (2) and (3) with $d = 39{,}022.78$ (from part (c))

we get Premium = $\underline{\underline{50{,}499.}}$