Solution of Homework # 3   171

3.3 $\because \begin{cases} S(0) = 1 \\ \lim_{x\to\infty} S(x) = 0 \end{cases}$ $\therefore$ $S(x) = e^{-x^3/12}$ can serve as a survival function

$$F(x) = 1 - S(x) = 1 - e^{-x^3/12}$$
$$f(x) = -S'(x) = \frac{1}{4}x^2 e^{-x^3/12}$$
$$\mu(x) = f(x)/S(x) = \frac{1}{4}x^2$$

3.4 a. $\int_0^\infty \mu(x)\,dx = \int_0^\infty (1+x)^{-3}dx = \frac{1}{2} < \infty$

b. $S(0) = S(3) = 0$ $S(x)$ is not monotonic over $[0, 3]$, that is $S'(x) > 0$ for some x

c. $\int_0^\infty f(x)\,dx = \int_0^\infty x^{n-1} e^{-\frac{x}{2}}\,dx \neq 1$

3.5 a. $\mu(x) = -S'(x)/S(x) = \frac{1}{100-x}$

b. $F(x) = 1 - S(x) = \frac{x}{100}$

c. $f(x) = -S'(x) = \frac{1}{100}$

d. $Pr(10 < X < 40) = S(10) - S(40) = 0.3$

3.7 a. ${}_{17}p_{19} = S(36)/S(19) = \frac{8}{9}$

b. ${}_{15}q_{36} = 1 - {}_{15}p_{36} = 1 - S(51)/S(36) = \frac{1}{8}$

c. ${}_{15|13}q_{36} = \dfrac{S(51) - S(64)}{S(36)} = \frac{1}{8}$

d. $\mu(36) = -S'(36)/S(36) = \frac{1}{128}$

e. $E[T(36)] = \int_0^{64} {}_tp_{36}\,dt = \int_0^{64} \frac{S(36+t)}{S(36)}\,dt$

$= \frac{128}{3}$

3.9 $\quad {}_{2|2}q_{20} = {}_2p_{20} \; {}_2q_{22} = \exp\left[-\int_{20}^{22} \mu(s)\,dt\right] \left[1 - \exp\left(\int_{22}^{24} \mu(s)\,dt\right)\right]$

$$= e^{-0.002}\left(1 - e^{-0.002}\right)$$

$$= 0.002$$