HOMEWORK (ONE-WAY ANOVA) **NAME:** ______________________

PROB. AND STAT. FOR ENG. (STAT:2020; BOGNAR) **SECTION:** ______________________

1. A watch maker wanted to compare four robotic milling machines for cut roughness. The roughness is measured in microns (1/1000 of a mm). After milling a number of parts on each machine, he summarized the data in the following table.

Mach 1	Mach 2	Mach 3	Mach 4
$n_1 = 5$	$n_2 = 5$	$n_3 = 6$	$n_4 = 6$
$\bar{x}_1 = 11.5$	$\bar{x}_2 = 8.9$	$\bar{x}_3 = 9.3$	$\bar{x}_4 = 12.2$
$s_1 = 1.3$	$s_2 = 1.5$	$s_3 = 1.0$	$s_4 = 1.1$

Assume the roughness for Machine i follows a $N(\mu_i, \sigma_i^2)$ distribution, $i = 1, 2, 3, 4$, and assume that $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4$.

(a) Find the mean squares between groups, $MS(Between)$.

$$\bar{x} = 10.5$$

$$SS(Between) = n_1(\bar{x}_1 - \bar{x})^2 + \cdots + n_k(\bar{x}_k - \bar{x})^2 = 43.78$$

$$MS(Between) = \frac{n_1(\bar{x}_1 - \bar{x})^2 + \cdots + n_k(\bar{x}_k - \bar{x})^2}{k-1} = \frac{43.78}{4-1} = 14.59$$

(b) Find the mean squares within groups, $MS(Within)$.

$$SS(Within) = (n_1 - 1)s_1^2 + \cdots + (n_k - 1)s_k^2 = 26.81$$

$$MS(Within) = \frac{SS(Within)}{n-k} = \frac{26.81}{22-4} = 1.489$$

(c) Test $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4$ vs H_a : not H_0 at the $\alpha = 0.05$ significance level using a 3-step one-way ANOVA test.

$$F^* = \frac{MS(Between)}{MS(Within)} = \frac{14.59}{1.489} = 9.80$$

$$F_{\alpha;k-1,n-k} = F_{0.05;3,18} = 3.16$$

We reject H_0.

(d) Find the $p-$value for the test in part (c). You will have to use the $F-$distribution web/phone applet to find the $p-$value.

$$p - value = P(F_{3,18} > 9.80) = 0.00047$$

Again, we reject H_0.

(e) Perform the Bonferroni pairwise comparison $H_0 : \mu_3 = \mu_4$ versus $H_a : \mu_3 \neq \mu_4$ at the α^* significance level.

$$\alpha^* = \frac{0.05}{\binom{4}{2}} = 0.00833$$

$$t^* = \frac{(\bar{x}_3 - \bar{x}_4) - (\mu_3 - \mu_4)_0}{\sqrt{MS(Within}\sqrt{\frac{1}{n_3} + \frac{1}{n_4}}} = -4.12$$

$$t_{\frac{\alpha^*}{2},n-k} = t_{0.004167,18} = 2.963$$

$$p - \text{value} = 2P(t_{(n-k)} > |t^*|) = 2P(t_{(18)} > 4.12) = 0.00064$$

(or $\in (0, 0.001)$ using table)

(f) Write out H_0 and H_a for the remaining 5 Bonferonni pairwise comparisons.
$H_0 : \mu_1 = \mu_2$ vs $H_a : \mu_1 \neq \mu_2$
$H_0 : \mu_1 = \mu_3$ vs $H_a : \mu_1 \neq \mu_3$
$H_0 : \mu_1 = \mu_4$ vs $H_a : \mu_1 \neq \mu_4$
$H_0 : \mu_2 = \mu_3$ vs $H_a : \mu_2 \neq \mu_3$
$H_0 : \mu_2 = \mu_4$ vs $H_a : \mu_2 \neq \mu_4$