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8
18
12
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40

Show your work on any problems that involve calculations.

Name: Solutions ―――――――――――――――――――――――― Course no. (30 or 105) ―――――

1. A sports statistician wishes to determine whether the brand of golf ball affects the distance that the ball travels after being hit. Ten professional golfers are asked to hit each of two brands of golf ball with their drivers. The golfers do not know which brand they are hitting, and the order in which they hit the balls is determined by flipping a coin. The distances that the balls travel are recorded.

To test for a difference in the mean distance traveled by each brand of golf ball, the statistician could use (circle the best answer):

(a) one-sample t test

(b) paired t test

(c) two-independent-sample t-test

(d) Chi-square test

(e) ANOVA

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2. A sociologist is studying whether there is a relationship between having children within the first three years of marriage and the divorce rate. From city marriage records, she selects a random sample of 400 couples who were married for the first time between 1995 and 2000, with both members of the couple between the ages of twenty and twenty-five. Of the 400 couples, 220 had at least one child within the first three years of marriage. Of the couples who had children, 83 were divorced within five years. Of the couples who didn't have children within three years, only 52 were divorced. Let p_1 represent the proportion divorced within five years among all couples who have a child within the first three years of marriage, and p_2 represent the proportion divorced within five years among all couples who do not have a child within the first three years.

(a) Represent these data in a two-by-two contingency table.

divorced

	Y	N	
child Y	83	137	220
N	52	128	180
5	135	265	400

(b) For testing the null hypothesis

$$H_0 : p_1 = p_2$$

what is the value of the pooled $\hat{p}$ – the point estimate of the value that both p_1 and p_2 would be equal to if the null hypothesis were true. Numeric answer; show your work.

$$\frac{135}{400} = 0.3375$$

3. A student is bored during a plane flight from Los Angeles to New York. He is curious about the proportion p of all passengers on his flight who are playing computer games. The plane has 416 seats, and all are taken. The student thinks that if he slowly walked through the plane, staring at every passenger and writing down whether they were playing computer games, people might get suspicious of him.

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Therefore the student randomly selects 25 seat numbers from the 416 seats on the 747. He walks through the plane and looks to see whether the person in each of his selected seats is playing computer games. He finds that 6 people are playing computer games, and 19 are not. He wants to use these results to compute a 95% confidence interval for p.

The student does not have access to statistical software during his flight, so he will have to calculate his confidence interval by hand.

(a) The population of interest to the student is (circle one):

    i. the 416 passengers on the plane

    ii. the 25 people whom he observes

    iii. all possible plane passengers

    iv. the proportion of the 25 passengers who are playing computer games

(b) Are the rules of thumb met so that the normal approximation method can be used to compute a confidence interval for p? Briefly explain.

Is the population at least 10 times as big as the sample? Yes! $416 > 10 \times 25$

Are there at least 10 successes and 10 failures in the sample? No. 6 yeses and 19 noes; It's ok if you use 5 instead of 10 as the criterion

(c) Calculate a 95% confidence interval for p using the normal approximation. (Numeric answer; show your work)

$$\hat{p} = \frac{6}{25} = .24$$

$$.24 \pm 1.96 \sqrt{\frac{.24(.76)}{25}} = (0.073, 0.407)$$

(d) Calculate a 95% confidence interval for p using the plus-four method.

$$\tilde{p} = \frac{6+2}{25+4} = .276$$

$$.276 \pm 1.96 \sqrt{\frac{0.276(1-0.276)}{29}} = (0.101, 0.451)$$

(e) What quantity should the student be 95% confident lies in his interval (circle on):

    i. the population proportion p

    ii. the sample proportion $\hat{p}$

    iii. both of the above

    iv. neither of the ablve

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18

4. The *Dietary Guidelines for Americans, 2010* recommend that Americans aged 2 and up consume less than 2,300 milligrams (mg) of sodium per day. That would be approximately 800 mg in each meal.

A nutritionist wishes to study whether school lunches served in elementary schools in her town average more than 800 mg of sodium. She will test the hypotheses

$$H_0: \quad \mu = 800$$
$$H_A: \quad \mu > 800$$

where μ represents the mean sodium level in all local elementary school lunches. She plans to randomly select 16 days over the next several months; on each of those days she will evaluate the sodium content of the school lunch menu for that day. Thus, her data will be 16 values of sodium content.

The nutritionist is quite sure that the population standard deviation σ of sodium in different school lunch menus is 120 mg. (It is unrealistic that she could know this, but assume it is correct in order to do the problem.)

(a) If the nutritionist wants to conduct her hypothesis test at significance level $\alpha = 0.05$, find the critical value of the sample mean $\bar{x}$. This is the value such that, if she gets an $\bar{x}$ this big or bigger, she will reject her null hypothesis. Numeric answer; show your work.

$$\sigma_{\bar{x}} = \frac{120}{\sqrt{16}} = 30$$

.05

$\mu_{\bar{x}} = 800$?

$\Longleftarrow \bar{x} \Longrightarrow$

5
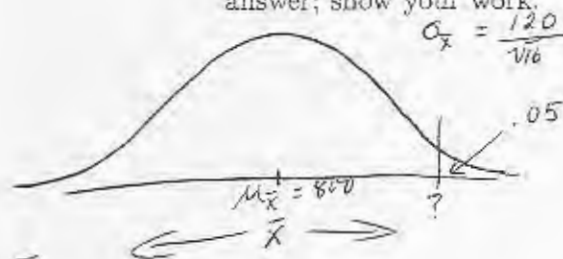
$$\bar{x} = \mu + \underbrace{1.645}_{z_{.95}} \sigma_{\bar{x}}$$
$$= 800 + 1.645 (30)$$
$$= 849.35$$

(b) The <u>power</u> of a hypothesis test is (circle on):

    i. the probability of rejecting the null hypothesis when it is true

    ii. the probability of not rejecting the null hypothesis when it is true

    iii. the probability of rejecting the null hypothesis when it is false

    iv. the probability of not rejecting the null hypothesis when it is false

    v. none of the above

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(c) The nutritionist believes that if the true μ is 900 mg or higher, action should be taken to change the school lunch menus. What is the power of her test if the true μ is 900? Numeric answer; show your work.

$$Z = \frac{849.35 - 900}{30} = -1.688$$

5    $Pr(Z > -1.688) = 1 - P(Z < -1.688) = 1 - .0457 = 0.954$

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(12)