22S:30/105
Statistical Methods and Computing

More Nonparametric Methods

Lecture 25
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The Wilcoxon Signed-Rank Test

- for single sample or paired samples
- useful when the population distribution is not normal and the sample size is not large
  - of the within-pair differences in paired sample case or of individual values in single sample case
- makes use of the magnitudes of the differences as well as their signs

Example

Kashima, Baker, and Landen (1988) studied whether media-based instruction could help the parents of mentally handicapped children become more effective at teaching their children self-help skills.

As part of the study, 17 families participated in a training program. Before and after the training program, the primary parent took the Behavioral Vignettes test, which assesses knowledge of behavioral modification principles. A higher score indicates greater knowledge.

The following are the pre- and post-test training scores for 12 of their families:

Pre	Post
7	11
6	14
10	16
16	17
8	9
13	15
8	9
14	17
16	20
11	12
12	14
13	15

May we conclude from these data that the training program increases knowledge of behavior modification principles? (We will test at the $\alpha = .01$ level.)

Hypotheses of the Wilcoxon Signed Rank Test

The null hypothesis is that, in the underlying population of differences among pairs, the median difference is equal to 0.

$$H_0 : M_d = 0$$

The alternative hypothesis may be one- or two-sided.

$$H_a : M_d > 0$$
$$H_a : M_d < 0$$
$$H_a : M_d \neq 0$$

If we define our differences as post - pre, then our alternative would be:

$$H_a : M_d > 0$$

Steps in the Wilcoxon signed-rank procedure

1. Select a random sample of n pairs of observations.
2. Compute the difference d_i in each pair of observations. Delete all pairs in which $d_i = 0$, and reduce n accordingly.
3. Ignoring the signs of the d_is, rank their absolute values from smallest to largest. When there are ties in absolute values, assign each tied value the mean of the rank positions the tied values occupy.
4. Assign to each rank the sign of the d_i that yields that rank.

5. Find T_+, the sum of the ranks with positive signs, and T_-, the sum of the ranks with negative signs.
6. Let the test statistic T equal the smaller of T_+ and T_-.

Pre- and Post-Test example

Pre	Post	d_i	Rank
7	11	4	9.5
6	14	8	12
10	16	6	11
17	16	-1	-2.5
8	9	1	2.5
13	15	2	6
8	9	1	2.5
14	17	3	8
16	20	4	9.5
11	12	1	2.5
12	14	2	6
13	15	2	6

The sum of the negative ranks is $T = 2.5$.

SAS for the Wilcoxon Signed Rank Test

- carried out automatically by `proc univariate`
- SAS computes a slightly different form of the test statistic

$$S = \Sigma(positive\ ranks) - \frac{n(n+1)}{4}$$

  recalling that n is the number of differences whose value is not equal to 0.
- computes p-value in two different ways depending on sample size
  - if $n \leq 20$, p-value is computed from each distribution of S, which can be enumerated under null hypothesis that distribution is symmetric around 0
  - when $n > 20$ approximate S is compared to approximate t distribution


```
data whatever ;
input pre post ;
diff = post - pre ;
datalines ;
 7  11
 6  14
10  16
16  17
 8   9
13  15
 8   9
14  17
16  20
11  12
12  14
13  15
;
run ;
```



```
proc univariate ;
var diff ;
run ;

             The UNIVARIATE Procedure
                  Variable:  diff

            Tests for Location: Mu0=0

 Test           -Statistic-    -----p Value------

 Student's t    t  4.521908    Pr > |t|    0.0009
 Sign           M         6    Pr >= |M|   0.0005
 Signed Rank    S        39    Pr >= |S|   0.0005
```

Note: For a 1-sided p-value, we would divide the 2-sided p-value by 2.

Interpreting the results

- Recall that we wanted to determine whether the audiovisual instruction improved parent's test scores.
- The null and alternative hypotheses regarding the median difference (that is, the median of post - pre), are

$$H_0 : M_d = 0$$
$$H_a : M_d > 0$$

- Can we reject H_0 at the .01 significance level?
- What does this mean with respect to the research question?

Sign Test in SAS

- Note that `proc univariate` also automatically carries out the sign test
- its version of sign test statistic is

$$M - \frac{n^+ - n^-}{2}$$

- use sign test if sample size is small and it is unreasonable to assume that population distribution is *symmetric*
- sign test p-value will often be a little larger than that of the Wilcoxon signed rank test (not so in this case)

The Wilcoxon Rank Sum Test

- used to compare nonparametrically two samples that have been drawn from independent populations
  - nonparametric analog of two-independent-sample t-test
- also called Mann-Whitney test, Mann-Whitney U test, and Mann-Whitney-Wilcoxon test

Assumptions of the Wilcoxon Rank Sum Test

- Two samples, of sizes n and m, have been drawn independently and randomly from their respective populations
- The measurement scale is at least ordinal
- The variable of interest is continuous
- If the populations differ, they differ only with respect to their medians
  - i.e., otherwise their shapes are approximately the same

Example: a question in pharmacokinetics

Is total plasma clearance of cefpiramide different in healthy people vs. patients with alcoholic cirrhosis?

Demotes-Mainard et al. (1991) measured total plasma clearance (ml/min) following a single 1-gram intravenous injection of cefpiramide in 10 healthy volunteers and 10 patients with alcoholic cirrhosis.

Case number	CIRR	CLEAR
1	0.000	21.700
2	0.000	29.300
3	0.000	25.300
4	0.000	22.800
5	0.000	21.300
6	0.000	31.200
7	0.000	29.200
8	0.000	28.700
9	0.000	17.200
10	0.000	25.700
11	1.000	14.600
12	1.000	18.100
13	1.000	12.300
14	1.000	8.800
15	1.000	10.300
16	1.000	8.500
17	1.000	29.300
18	1.000	8.100
19	1.000	6.900
20	1.000	7.900

Can we conclude at the $\alpha = .01$ significance level that median clearance rate is different in healthy patients vs. those with alcoholic cirrhosis?

Hypotheses for the Wilcoxon Rank Sum Test

$$H_0 : M_1 = M_2$$

The alternative hypothesis may be one- or two-sided.

$$H_a : M_1 > M_2$$
$$H_a : M_1 < M_2$$
$$H_a : M_1 \neq M_2$$

Procedure for the Wilcoxon Rank Sum Test

- Combine the two samples into one large group, and sort values from smallest to largest.
- Rank the values. When there are ties in absolute values, assign each tied value the mean of the rank positions the tied values occupy.
- Sum the ranks *within each original sample*
- The test statistic is W, the smaller of the two sums.

Ranked values for clearance example

Case number	CIRR	CLEAR	
1	0.000	21.700	12.0
2	0.000	29.300	18.5
3	0.000	25.300	14.0
4	0.000	22.800	13.0
5	0.000	21.300	11.0
6	0.000	31.200	20.0
7	0.000	29.200	17.0
8	0.000	28.700	16.0
9	0.000	17.200	9.0
10	0.000	25.700	15.0
11	1.000	14.600	8.0
12	1.000	18.100	10.0
13	1.000	12.300	7.0
14	1.000	8.800	5.0
15	1.000	10.300	6.0
16	1.000	8.500	4.0
17	1.000	29.300	18.0
18	1.000	8.100	3.0
19	1.000	6.900	1.0
20	1.000	7.900	2.0

The Wilcoxon Rank Sum test in SAS

- use `proc npar1way`

```
data clear ;
input id cirr clear ;
datalines ;
        1       0.000   21.700
        2       0.000   29.300
        3       0.000   25.300
        4       0.000   22.800
        5       0.000   21.300
        6       0.000   31.200
        7       0.000   29.200
        8       0.000   28.700
        9       0.000   17.200
        10      0.000   25.700
        11      1.000   14.600
        12      1.000   18.100
        13      1.000   12.300
        14      1.000   8.800
        15      1.000   10.300
        16      1.000   8.500
        17      1.000   29.300
        18      1.000   8.100
        19      1.000   6.900
        20      1.000   7.900
;
run ;
```



```
proc npar1way wilcoxon ;
class cirr ;
var clear ;
run ;

                        The NPAR1WAY Procedure

            Wilcoxon Scores (Rank Sums) for Variable clear
                     Classified by Variable cirr

                     Sum of      Expected       Std Dev          Mean
 cirr       N        Scores      Under H0      Under H0         Score
 --------------------------------------------------------------------
 0         10        145.50         105.0     13.223782        14.550
 1         10         64.50         105.0     13.223782         6.450

                    Average scores were used for ties.

                         Wilcoxon Two-Sample Test

                     Statistic             145.5000

                     Normal Approximation
                     Z                       3.0249

                     One-Sided Pr >  Z       0.0012
                     Two-Sided Pr > |Z|      0.0025

                     t Approximation
                     One-Sided Pr >  Z       0.0035
                     Two-Sided Pr > |Z|      0.0070

                Z includes a continuity correction of 0.5.
```


The Kruskal Wallis Test

- named after William Kruskal and W. Allen Wallis
- non-parametric method for testing equality of population medians among groups
- like a one-way analysis of variance with the data replaced by their ranks
- extension of the Wilcoxon rank sum test to 3 or more groups
- performed in SAS by `proc npar1way`