22S:30/105 Statistical Methods and Computing

Introduction to Nonparametric Methods

Lecture 24
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Parametric methods

- based on the assumption that the population(s) from which our samples are drawn follow a distribution, the general form of which is known
  - e.g. normal or binomial
- research interest is in estimating, or testing a hypothesis about, one or more population parameters
- examples: z tests, t tests, and ANOVA for making inference about means of populations assumed to be normal

"Nonparametric" or "distribution-free" statistical methods

- allow for testing hypotheses that are not statements about population parameter values
- may be used when the form of the distribution of the sampled population is unknown
- can be used when data being analyzed consist merely of rankings or classifications
  - i.e. when arithmetic operations required for parametric procedures cannot be done
  - example: data on patient conditions reported as "better," "same," or "worse"

Example for the Sign Test

- We wish to compare the effectiveness of two ointments (A, B) in reducing sunburn in people whose skin is sensitive to sunlight.
- For each person in the study, we randomly select either the left arm or the right arm and apply ointment A. We then apply ointment B to the same area of the other arm.
- We then expose the person to 1 hour of sunlight and compare the two arms with respect to degree of redness.
- We can make only the following qualitative assessments:
  1. "A" arm is not as red as "B" arm.
  2. "A" arm is redder than "B" arm.
  3. Arms are equally red.

How might we compare the ffectiveness of the two ointments *if we were able to measure redness on a quantitative scale*?

In the situation described here, we cannot observe the actual values of within-person differences in redness betweem the A arm and the B arm.

What we can observe are the *signs* of the differences:

1. "A" arm is not as red as "B" arm (+)
2. "A" arm is redder than "B" arm (-)
3. Arms are equally red (0)

The Sign Test

The null hypothesis of the sign test is that in the underlying population of differences, the median difference M is 0.

$$H_0 : M = 0.$$

The alternative hypothesis may be either one-sided or two-sided.

$$H_0 : M > 0$$

$$H_0 : M < 0$$

$$H_0 : M \neq 0$$

To carry out the sign test:

- Ignore the pairs (or observations) with difference of 0.
- Denote the number of remaining pairs as n.
- Count the number of plus signs, and denote it D.
- Note that under the null hypothesis, we would expect approximately equal numbers of plus and minus signs.
  - more precisely, under the null hypothesis, D follows a binomial distribution with success probability $p = 1/2$ and number of trials n
  - This binomial distribution has

$$mean = np = \frac{n}{2}$$

$$standarddeviation = \sqrt{np(1-p)} = \sqrt{\frac{n}{4}}$$

- We must evaluate how likely we would have been to obtain a value of D as extreme as what we got, or more extreme, if the null is true.
- Your textbook gives the test statistic for use with a normal approximation to the binomial distribution. This is appropriate for use if $n \geq 20$. The value is compared to the standard normal distribution.
- Otherwise, we will use the binomial distribution directly.

The sign test for the skin ointment data

We wish to do a two-sided test, i.e.

$$H_a : M \neq 0$$

at the $\alpha = .05$ significance level.

The results for 45 subjects are:

1. 22 people had the "A" arm less red (+)
2. 18 people had the "B" arm less red (-)
3. 5 people had no difference (0)

- n = 45 - 5 = 40
- D = 22
- normal approximation is valid because $n \geq 20$.

$$z_+ = \frac{D - (n/2)}{\sqrt{n/4}}$$

$$= \frac{22 - 20}{\sqrt{10}}$$
$$= 0.632$$

For a 2-sided test, we must compare this value to the .025 cutoff for the standard normal distribution, which is 1.96.

Because $0.632 < 1.96$, we cannot reject H_0.

Equivalently, we can determine the p-value of our test by finding $P(z > 0.632) =\approx .264$.

- This would be the p-value for a 1-sided test.
- To find the p-value for our 2-sided test, we multiply by 2.

$$p = 2(.264) = .528 > \alpha = .05$$

So again, we cannot reject H_0. We conclude that the data do not provide evidence that one ointment is better than the other.

The sign test with small sample size

Suppose that instead of 40 patients with non-zero differences, we had had

1. 5 people had the "A" arm less red (+)
2. 3 people had the "B" arm less red (-)
3. 37 people had no difference (0)

Then

- n = 45 - 37 = 8
- D = 5
- normal approximation is inappropriate because $n < 20$.
  - we will do exact calculation of the p-value using the binomial distribution

Because $D > n/2 = 4$, we will compute

$$\begin{aligned} P(D \geq 5|H_0) &= P(D=5) + P(D=6) \\ &\quad + P(D=7) + P(D=8) \\ &= .2188 + .1094 + .0313 + .0039 \\ &= 0.3634 \end{aligned}$$

This is a one-sided p-value. We must multiply by 2 to get the approximate 2-sided p-value.

$$2(0.3634) = 0.7268 > .05$$

So again we would not reject H_0.

More on the sign test

- Can be used with single-sample or paired-sample problems
- Frees us from having to make any assumptions about the underlying distribution of differences
- If we have any information about the magnitude of the individual differences, the sign test wastes it.