

**22S:30/105**  
**Statistical Methods and**  
**Computing**

**More Nonparametric Methods**

Lecture 25  
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**Example**

Kashima, Baker, and Landen (1988) studied whether media-based instruction could help the parents of mentally handicapped children become more effective at teaching their children self-help skills.

As part of the study, 17 families participated in a training program. Before and after the training program, the primary parent took the Behavioral Vignettes test, which assesses knowledge of behavioral modification principles. A higher score indicates greater knowledge.

**The Wilcoxon Signed-Rank Test**

- for single sample or paired samples
- useful when the population distribution is not normal and the sample size is not large
  - of the within-pair differences in paired sample case or of individual values in single sample case
- makes use of the magnitudes of the differences as well as their signs

The following are the pre- and post-test training scores for 12 of their families:

Pre	Post
7	11
6	14
10	16
16	17
8	9
13	15
8	9
14	17
16	20
11	12
12	14
13	15

May we conclude from these data that the training program increases knowledge of behavior modification principles? (We will test at the  $\alpha = .01$  level.)

## Hypotheses of the Wilcoxon Signed Rank Test

The null hypothesis is that, in the underlying population of differences among pairs, the median difference is equal to 0.

$$H_0 : M_d = 0$$

The alternative hypothesis may be one- or two-sided.

$$H_a : M_d > 0$$

$$H_a : M_d < 0$$

$$H_a : M_d \neq 0$$

If we define our differences as post - pre, then our alternative would be:

$$H_a : M_d > 0$$

5. Find  $T_+$ , the sum of the ranks with positive signs, and  $T_-$ , the sum of the ranks with negative signs.

6. Let the test statistic  $T$  equal the smaller of  $T_+$  and  $T_-$ .

## Steps in the Wilcoxon signed-rank procedure

1. Select a random sample of  $n$  pairs of observations.
2. Compute the difference  $d_i$  in each pair of observations. Delete all pairs in which  $d_i = 0$ , and reduce  $n$  accordingly.
3. Ignoring the signs of the  $d_i$ s, rank their absolute values from smallest to largest. When there are ties in absolute values, assign each tied value the mean of the rank positions the tied values occupy.
4. Assign to each rank the sign of the  $d_i$  that yields that rank.

## Pre- and Post-Test example

Pre	Post	d_i	Rank
7	11	4	9.5
6	14	8	12
10	16	6	11
17	16	-1	-2.5
8	9	1	2.5
13	15	2	6
8	9	1	2.5
14	17	3	8
16	20	4	9.5
11	12	1	2.5
12	14	2	6
13	15	2	6

The sum of the negative ranks is  $T = 2.5$ .

## SAS for the Wilcoxon Signed Rank Test

- carried out automatically by `proc univariate`
- SAS computes a slightly different form of the test statistic

$$S = \Sigma(\text{positive ranks}) - \frac{n(n+1)}{4}$$

recalling that  $n$  is the number of differences whose value is not equal to 0.

- computes p-value in two different ways depending on sample size
  - if  $n \leq 20$ , p-value is computed from each distribution of  $S$ , which can be enumerated under null hypothesis that distribution is symmetric around 0
  - when  $n > 20$  approximate  $S$  is compared to approximate t distribution

```
data whatever ;
input pre post ;
diff = post - pre ;
datalines ;
7 11
6 14
10 16
16 17
8 9
13 15
8 9
14 17
16 20
11 12
12 14
13 15
;
run ;
```

```
proc univariate ;
var diff ;
run ;
```

The UNIVARIATE Procedure  
Variable: diff

Tests for Location: Mu0=0

Test	-Statistic-	-----p Value-----
Student's t	t 4.521908	Pr >  t  0.0009
Sign	M 6	Pr >=  M  0.0005
Signed Rank	S 39	Pr >=  S  0.0005

Note: For a 1-sided p-value, we would divide the 2-sided p-value by 2.

## Interpreting the results

- Recall that we wanted to determine whether the audiovisual instruction improved parent's test scores.
- The null and alternative hypotheses regarding the median difference (that is, the median of post - pre), are

$$H_0 : M_d = 0$$

$$H_a : M_d > 0$$

- Can we reject  $H_0$  at the .01 significance level?

- What does this mean with respect to the research question?

## Sign Test in SAS

- Note that `proc univariate` also automatically carries out the sign test
- its version of sign test statistic is

$$M - \frac{n^+ - n^-}{2}$$

- use sign test if sample size is small and it is unreasonable to assume that population distribution is *symmetric*
- sign test p-value will often be a little larger than that of the Wilcoxon signed rank test (not so in this case)

## The Wilcoxon Rank Sum Test

- used to compare nonparametrically two samples that have been drawn from independent populations
  - nonparametric analog of two-independent-sample t-test
- also called Mann-Whitney test, Mann-Whitney U test, and Mann-Whitney-Wilcoxon test

## Assumptions of the Wilcoxon Rank Sum Test

- Two samples, of sizes  $n$  and  $m$ , have been drawn independently and randomly from their respective populations
- The measurement scale is at least ordinal
- The variable of interest is continuous
- If the populations differ, they differ only with respect to their medians
  - i.e., otherwise their shapes are approximately the same

## Example: a question in pharmacokinetics

Is total plasma clearance of cefpiramide different in healthy people vs. patients with alcoholic cirrhosis?

Demotes-Mainard et al. (1991) measured total plasma clearance (ml/min) following a single 1-gram intravenous injection of cefpiramide in 10 healthy volunteers and 10 patients with alcoholic cirrhosis.

Case number	CIRR	CLEAR
1	0.000	21.700
2	0.000	29.300
3	0.000	25.300
4	0.000	22.800
5	0.000	21.300
6	0.000	31.200
7	0.000	29.200
8	0.000	28.700
9	0.000	17.200
10	0.000	25.700
11	1.000	14.600
12	1.000	18.100
13	1.000	12.300
14	1.000	8.800
15	1.000	10.300
16	1.000	8.500
17	1.000	29.300
18	1.000	8.100
19	1.000	6.900
20	1.000	7.900

Can we conclude at the  $\alpha = .01$  significance level that median clearance rate is different in healthy patients vs. those with alcoholic cirrhosis?

### Procedure for the Wilcoxon Rank Sum Test

- Combine the two samples into one large group, and sort values from smallest to largest.
- Rank the values. When there are ties in absolute values, assign each tied value the mean of the rank positions the tied values occupy.
- Sum the ranks *within each original sample*
- The test statistic is  $W$ , the smaller of the two sums.

### Hypotheses for the Wilcoxon Rank Sum Test

$$H_0 : M_1 = M_2$$

The alternative hypothesis may be one- or two-sided.

$$H_a : M_1 > M_2$$

$$H_a : M_1 < M_2$$

$$H_a : M_1 \neq M_2$$

Ranked values for clearance example

Case number	CIRR	CLEAR	rank
1	0.000	21.700	12.0
2	0.000	29.300	18.5
3	0.000	25.300	14.0
4	0.000	22.800	13.0
5	0.000	21.300	11.0
6	0.000	31.200	20.0
7	0.000	29.200	17.0
8	0.000	28.700	16.0
9	0.000	17.200	9.0
10	0.000	25.700	15.0
11	1.000	14.600	8.0
12	1.000	18.100	10.0
13	1.000	12.300	7.0
14	1.000	8.800	5.0
15	1.000	10.300	6.0
16	1.000	8.500	4.0
17	1.000	29.300	18.0
18	1.000	8.100	3.0
19	1.000	6.900	1.0
20	1.000	7.900	2.0

## The Wilcoxon Rank Sum test in SAS

- use `proc npar1way`

```
data clear ;
input id cirr clear ;
datalines ;
  1      0.000  21.700
  2      0.000  29.300
  3      0.000  25.300
  4      0.000  22.800
  5      0.000  21.300
  6      0.000  31.200
  7      0.000  29.200
  8      0.000  28.700
  9      0.000  17.200
 10      0.000  25.700
 11      1.000  14.600
 12      1.000  18.100
 13      1.000  12.300
 14      1.000   8.800
 15      1.000  10.300
 16      1.000   8.500
 17      1.000  29.300
 18      1.000   8.100
 19      1.000   6.900
 20      1.000   7.900
;
run ;
```

```
proc npar1way wilcoxon ;
class cirr ;
var clear ;
run ;
```

```

                                The NPARIWAY Procedure

                                Wilcoxon Scores (Rank Sums) for Variable clear
                                Classified by Variable cirr

                                -----
                                cirr      N      Sum of      Expected      Std Dev      Mean
                                -----      Scores      Under H0      Under H0      Score
                                -----
                                0          10      145.50      105.0         13.223782     14.550
                                1          10       64.50      105.0         13.223782     6.450
                                -----

                                Average scores were used for ties.

                                Wilcoxon Two-Sample Test

                                Statistic              145.5000

                                Normal Approximation
                                Z                          3.0249

                                One-Sided Pr > Z          0.0012
                                Two-Sided Pr > |Z|         0.0025

                                t Approximation
                                One-Sided Pr > Z          0.0035
                                Two-Sided Pr > |Z|         0.0070

```

Z includes a continuity correction of 0.5.

## The Kruskal Wallis Test

- named after William Kruskal and W. Allen Wallis
- non-parametric method for testing equality of population medians among groups
- like a one-way analysis of variance with the data replaced by their ranks
- extension of the Wilcoxon rank sum test to 3 or more groups
- performed in SAS by `proc npar1way`