22S:166
Homework due Mon. 10/09/06

NOTE: Save your program for question 2, as you will be adding to it in a future homework assignment.

1. The function

   $$x^3 + 4x^2 - 10$$

   has a real root in [0,3].

   Use the built-in R function called `uniroot` to locate this root.

2. EM algorithm for grouped exponential data.

   Suppose that 50 machines were simultaneously put into operation in an adverse environment. Once a week for four weeks, an engineer was sent in to observe how many machines had failed since the last check. Any time a machine failed, it was removed and not replaced. At the end of the 4-week test, the following report was issued:

Time interval in days a_j - b_j	Number failing in interval n_j
0 - $<$ 8	15
8 - $<$ 15	9
15 - $<$ 22	7
22-$<$ 28	6
$\geq$ 28	13

   We believe that the exact failure times are drawn from an exponential distribution; that is

   $$f(t) = \lambda\ exp(-\lambda t), \quad t > 0$$

   Obtain the maximum likelihood estimate of λ, based on the grouped data. Proceed as follows.

   (a) Determine the complete-data sufficient statistic for λ, that is, the sufficient statistic if all the exact failure times were known.

   (b) Find the complete-data maximum likelihood estimator of λ as a function of the sufficient statistic (the expression, not the number).

   (c) Find a closed form expression for the conditional expectation of the exact failure time of an observation known to lie in the interval a_j - b_j, conditional on $\lambda = \lambda^{(k)}$.

   (d) Use the results of the previous parts to code an EM algorithm to compute the mle of λ (the numeric estimate).

   (e) Choose a stopping criterion and run your algorithm. Report your maximum likelihood estimate for λ. Turn in your code and output.

3. Verify that

   $$log[p(\theta|Y)] = log[p(\theta|Z,Y)] - log[p(Z|\theta,Y)] + log[p(Z|Y)]$$

   (This is the first step in proving that every EM algorithm increases the posterior (or likelihood) $p(\theta|Y)$ at each iteration.