

22S:138
Specifying priors on variance
components
Intro to Generalized Linear Models

Lecture 17
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Possible approaches to specifying priors on variance components

- fit a frequentist mixed model to a different but related dataset and get estimated variances of slopes and intercepts
- use results of someone else's study
- elicit expert opinion
- use common sense

Priors on variance components

- stage 2, formulation 1

$$\alpha_{0i} | \beta_0, \tau_{\alpha_0}^2 \sim N(\beta_0, \tau_{\alpha_0}^2)$$

$$\alpha_{1i} | \beta_1, \tau_{\alpha_1}^2 \sim N(\beta_1, \tau_{\alpha_1}^2)$$

- stage 2, multivariate formulation

$$\begin{bmatrix} \alpha_{0i} \\ \alpha_{1i} \end{bmatrix} \mid \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} \Sigma_{\alpha} \sim N_2 \left(\begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}, \Sigma_{\alpha}^{-1} \right)$$

- $\tau_{\alpha_0}^2$ is precision of subject-specific intercepts
- $\tau_{\alpha_1}^2$ is precision of subject-specific slopes

Common-sense approach

- What do intercepts mean?
 - birthweight of baby rat in grams
 - $\sqrt[4]{CD4count}$ at study entry
- What would be a plausible but very small value for this quantity?
- Plausible but unusually large value?
- (large - small) \simeq 4 standard deviations
- baby rats example
 - guess $\sigma_{\alpha_0} = 10$
 - guess $\sigma_{\alpha_1} = \sqrt{0.1}$
 - corresponding single number guess for $\tau_{\alpha_0}^2$
 - corresponding single number guess for $\tau_{\alpha_1}^2$

- How would you use this to construct Gamma priors?

- How did Gelfand *al.* use this to construct Wishart prior for second formulation?

$$\begin{aligned}
 R &= \Sigma_{\alpha}^{-1} \\
 R &\sim \text{dwish}(\text{Omega}, 2) \\
 \text{Omega} &= \begin{bmatrix} 200 & 0 \\ 0 & 0.2 \end{bmatrix} \\
 \text{Omega}^{-1} &= \begin{bmatrix} 1/200 & 0 \\ 0 & 1/0.2 \end{bmatrix} \\
 E(R^{-1}) &= 2 \text{Omega}^{-1} \\
 &= \begin{bmatrix} 1/100 & 0 \\ 0 & 1/0.1 \end{bmatrix}
 \end{aligned}$$

- Better to have larger degrees of freedom and use procedure we outlined in previous lecture

Generalized linear models (GLMs)

- goal is to determine relationship between predictor variables and a response variable that is not normally distributed
 - may not even be quantitative
- example: vote on library proposal in Iowa City 2 years ago
 - subjects are individual voters
 - response variables Y_i are whether person voted for or against (coded 1 and 0)
 - predictors might be gender, income level, whether person has children, etc.

Centering covariates and priors on variance components

- correlations between α_{i0} 's and α_{i1} 's
- even if covariate is centered, random effects do not become independent in HNLM
- centering is straightforward if all subjects have the same set of values of predictor variable
- question: if subjects have different values of the predictor, should we center around
 - subject-specific average ($x_{ij} - x.\text{bar}_i$)
 - overall average ($x_{ij} - x.\text{bar}$)
- centering affects the priors on
 - β_0
 - $\tau_{\alpha_0}^2$ or Σ_{α}

Why normal linear regression would not work for example

- $$E(Y_i) = \beta_0 + \beta_1 \text{income}_i$$
 - there would always be values of income for which results would be negative or > 1
 - residuals could not be normally distributed since real Y_i 's are all 0's and 1's

Solution: logistic regression

- (special case of GLM)
- Y_i 's actually have what distribution
- $E(Y_i) =$
- Choose an appropriate transformation of $E(Y_i)$ that can take on values anywhere on real line
- one good choice is logit

$$\text{logit}(p_i) = \log\left(\frac{p_i}{1 - p_i}\right)$$

- model *transformed* expectations as linear function of predictors

$$\text{logit}(p_i) = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \dots$$

- there is no separate variance parameter at the level of the likelihood (why)

- so likelihood is completely specified in WinBUGS as

```
Y[i] ~ dbern(p[i])
logit(p[i]) <- beta[0] + beta[1] * x[i,1]
```

and priors are needed only on β s

more general case of GLM

- conditional on model parameters and values of predictor variables, response variables Y_i are independent draws from distributions in a natural exponential family
- we are most interested in 2 natural exponential families
 - bernoulli (or binomial)
 - Poisson
- both are 1-parameter families
- $E(Y_i) = g^{-1}(\beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i})$ where g is a link function
 - monotonic and differentiable
 - log is most commonly used link function for Poisson regression