22S:138, Bayesian Statistics
Fall 2007, Homework 7

Due: Mon., 10/29 in class

1. The exponential distribution is commonly used to model data on time from some starting point until an event occurs. The probability density function for a random variable Y drawn from an exponential distribution with parameter λ is

$$p(y \mid \lambda) \;=\; \lambda\, e^{-\lambda y}, \;\; 0 < y < \infty$$

Researchers wish to compare 5 medications that reduce fever. These medications are all different formulations of the same drug (capsule, liquid, tablet, etc.). The experiment involves 30 patients, each with a temperature of at least 102 degrees. Six patients are randomly assigned to medication 1, six patients to medication 2, etc. After the patient takes the medication assigned to him, his temperature is monitored continuously. The outcome variable of interest is the time in hours from when the medication is taken until the patient's temperature goes below 99.5 degrees.

Let y_{ij} denote this time for patient j who received medication i.

The researchers will carry out a Bayesian analysis. They assume that all the times for patients receiving the same medication, say i, follow an exponential distribution with the same parameter λ_i. They believe that the λs for different treatments are different, but they have no idea of which ones will be larger or smaller.

Below is WinBUGS code for fitting a 3-stage hierarchical model to their data.

```
model {
for (i in 1:N) {
     for (j in 1:M) {
          Y[i,j] ~ dexp( lambda[i])
                }
    lambda[i] ~ dgamma( alpha, beta)
}
alpha ~ dgamma( 1,1)
beta ~ dgamma(1,1)
overall <- alpha / beta
}
```

   (a) Draw a directed graph for this model. You may either do this by hand or use DoodleBUGS or any graphing software that you know.

   (b) Derive expressions proportional to the full conditional distributions for lambda[i], alpha, and beta. If any of them are standard distributions, identify them by family and parameters. If any of them are not standard distributions, say so.

2. Suppose another level were added to the hierarchy as follows:


```
model {
for (i in 1:N) {
     for (j in 1:M) {
          Y[i,j] ~ dexp( lambda[i])
                }
    lambda[i] ~ dgamma( alpha, beta)
}

alpha ~ dgamma( a1, b1)
beta ~ dgamma(a2, b2 )

a1 ~ dexp(2)
b1 ~ dexp(2)
a2 ~ dexp(2)
b2 ~ dexp(2)

overall <- alpha / beta
}
```

   (a) Will the full conditionals for the lambda[i]s change? Why or why not?

   (b) Derive expressions proportional to the full conditionals for alpha, beta, a1, and b1. If possible, identify them as standard distributions.