

22S:138, Bayesian Statistics  
Fall 2007, Homework 6

Due: Mon., 10/22 in class

Consider the “Dyes” example in Examples 1 under the WinBUGS help menu.

1. Under this model, are all the  $y_{ij}$ 's (the yields for all the different samples in different batches) considered exchangeable? Why or why not?
2. Under this model, are all the  $y_{2j}$ 's (the yields for all the samples in batch 2) considered exchangeable?
3. Under this model, are the  $\mu_i$ 's considered exchangeable?
4. Run three parallel chains for 15,000 iterations to fit the model as given in the example, with the *third prior* (the one that is *not* recommended).
  - For one chain, use the initial values provided in the example.
  - For another chain, use  
`list( mu = c( 1525, 1525, 1525, 1525, 1525, 1525 ), theta = 1525, tau.btw = 10000, tau.with = .001)`
  - For another chain, set the mu's equal to values that are very different from each other and from theta, and reverse the values of tau.between and tau.within given previously.
  - Monitor  $\theta$ ,  $\mu$ ,  $\tau_{between}^2$ ,  $\tau_{within}^2$ ,  $\sigma_{between}^2$ , and  $\sigma_{within}^2$ , beginning with the first iteration.
  - Obtain autocorrelation plots, history plots, Gelman and Rubin diagnostics, and output statistics. Look at plots based on each chain individually as well as on the combined chains. You do NOT have to print and turn in all the plots! Choose a few typical ones to print and comment on.
5. Run another 15000 iterations and obtain the same output. Do the autocorrelation plots change much with additional iterations? Does the Monte Carlo error decrease with additional iterations?
6. The use of very vague priors on both of the variance components, as shown in the example, is a bad idea. If an improper prior is placed on  $\tau_{between}$ , the posterior will be improper as well. The priors on  $\tau_{within}$  and  $\tau_{between}$  are so vague that this is *almost* the case here. That is the reason why so many iterations were required in the sampler run.

Replace the vague prior on  $\tau_{between}$  with an informative prior. Choose its parameters this way. You want an *inverse gamma* prior on  $\sigma_{between}^2$  (the *variance* of batch means) that has a mean of 2000 and a variance of 250,000. Find the correct values of the parameters  $\alpha$  and  $\beta$ . Now in WinBUGS, put a *gamma* prior with the same values of  $\alpha$  and  $\beta$  on  $\tau_{between}$  (the *precision*).

7. Repeat steps (4) and (5) above with your modified model, answering the same questions. In addition, compare the autocorrelation plots and Monte Carlo errors between the original model and the model with the informative prior. (Again, you don't have to print out many of the plots. Summarize the comparison in a few sentences.)

8. Repeat step (4):

- using Prior 1 in the example
- using Prior 2 in the example

Again, summarize what you see in history plots, autocorrelation plots, and BGR diag plots.