Name: ___________________________________

Bayesian Statistics, 22S:138
PRACTICE PROBLEMS FOR MIDTERM 1, 2006
Midterm 1, Fall 2005

Show any computations that you carry out. Use the back of your exam paper if you run out of space. Point values for each question are shown in parentheses.

1. A die is a cube with 6 faces, each showing one of the integers from 1 to 6. If the die is fair, when it is rolled, each face is equally likely to end up on top.

   An experiment consists of rolling a fair die once. Let event A be "the face on top has an even number." Let event B be "the number on the face on top is divisible by 3."

   (a) (2) List out the sample space of the experiment.

   (b) (3) List the outcomes in B, and find $P(B)$.

   (c) (3) list the outcomes in $A \cap B$ and find $P(A \cap B)$.

   (d) (5) Are the events A and B independent? Show why or why not.

2. An engineer is studying a particular type of steel I beam. She wishes to infer about the population mean μ of the distance that this type of beam will sag under a standard load. She believes that the distribution of the amount of sag in individual I beams of this type is normal with unknown mean μ and known standard deviation $\sigma = 0.5$ mm. (Known population standard deviation is `not` a reasonable assumption, of course.)

   She takes a sample of $n = 5$ I beams and measures the distance $y_i,\ i = 1, \ldots, 5$ that each one sags under the standard load.

   (a) The engineer places a normal prior on μ with prior mean 5 mm and prior variance 0.25 mm^2. The sample mean $\bar{y}$ of her data is 4.91 mm.

      i. (5) Find the posterior distribution $p(\mu|\bar{y})$. Give the name of the family of distributions, and the numeric value(s) of its parameter(s).

      ii. (3) The engineer wishes to do a Bayesian test of the hypotheses

$$H_0: \ \mu \ \leq \ 5.20\ mm$$
$$H_A: \ \mu \ > \ 5.20\ mm$$

      Write the R function that you would use to evaluate the posterior probability that H_0 is true.

      iii. (4) Find the posterior predictive distribution $p(y_{new}|y)$ for the amount of sag in a future steel I beam of this type. Name the family and give its parameter(s).

(b) (5) Suppose that the engineer had placed a *discrete* prior on μ as follows:

$$Pr(\mu = m) = \begin{cases} \frac{1}{3}, & m = 4.80,\ 5.20,\ 5.40 \\ 0, & otherwise \end{cases}$$

Find the posterior distribution. If it is continuous, name the parametric family and give numerical value(s) of its parameter(s). If it is discrete, state the values of μ with non-zero posterior probability and show how to calculate the posterior probability of each. (You don't have to actually calculate the probabilities; just write out the formula with all the correct numbers filled in.)

3. The data value x is a random draw from a probability distribution depending on a parameter δ:

$$p(x|\delta) \propto \delta^2 x e^{-\delta x} \quad 0 < x < \infty$$
$$\delta > 0$$

(a) (3) What parametric family is this? (Name and parameter(s)).

(b) (3) To what parametric family of distributions would the conjugate prior for δ belong? (Just name the family.)

4. For each of the descriptions of data below, circle the name of the parametric distribution that is most likely to be appropriate for the likelihood.

   (a) (2) Researchers choose a simple random sample of counties from amongst all the counties in the U.S. The variable reported for each of the selected counties is the number of cases of detached retina (a non-contagious eye problem) per 1000 population in the year 2003.

      i. binomial
      ii. gamma
      iii. normal
      iv. poisson

   (b) (2) Researchers choose a simple random sample of cats of a particular variety. The variable measured on each cat is head circumference in inches.

      i. binomial
      ii. gamma
      iii. normal
      iv. poisson