22S:138
Bayesian Statistics

Lecture 8
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Posterior predictive distribution of a future observation

- $p(y_{new}|y)$ is Normal
  - mean is posterior mean
  - variance is sum of
    - data variance σ^2 (assumed known in this unrealistic case)
    - variance of posterior mean

Jeffreys prior for normal mean with data variance assumed known

- $p(\mu) \propto 1, \quad -\infty < \mu < \infty$
- limit of $N(\mu_0, \sigma_0^2)$ as σ_0^2 goes to ∞
  σ_0^2 is prior variance
- equivalently, limit of $N(\mu_0, \tau_0^2)$ as τ_0^2 goes to 0
  τ_0^2 is prior precision

What is the posterior predictive distribution of the healing rate of a new newt?

Inference about the spread of a normal distribution

- primary research question may concern variability of response variable in population
  - quality control in industry
  - reponse to medical treatment

Inference for the variance of a normal distribution

- Suppose in the newt healing rate example that we knew the population mean $\mu = 25$ but we did not know the population variance σ^2
- Equivalently, we don't know the population precision τ^2
- We wish to infer about the distributions of these parameters that describe the spread of the normal distribution.

- Recall the joint sampling distribution of n observations modelled as conditionally independent draws from a normal

$$p(y_1, \ldots, y_n|\mu, \sigma^2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} exp\left[-\frac{(y_i - \mu)^2}{2\sigma^2}\right]$$
$$\propto \frac{1}{(\sigma^2)^{\frac{n}{2}}} exp\left[-\frac{\Sigma_{i=1}^{n}(y_i - \mu)^2}{2\sigma^2}\right]$$

- We will assume (unrealistically) that μ is a known constant

Sufficient statistic

- sufficient statistic for σ^2 is $\Sigma(y_i - \mu)^2$
- we can write likelihood equivalently as

$$p(y|\sigma^2) \propto \frac{1}{(\sigma^2)^{\frac{n}{2}}} exp\left[-\frac{nv}{2\sigma^2}\right], \ 0 < \sigma^2 < \infty$$

where

$$v = \frac{1}{n}\Sigma(y_i - \mu)^2$$

- What is corresponding conjugate prior?

Inverse gamma distribution

What is posterior?

$$p(\sigma^2|y) \ \propto ?$$

Alternative parameterization of prior

$$p(\sigma^2) \ = \ IG(\frac{\nu_0}{2}, \frac{\nu_0\sigma_0^2}{2})$$

Then we can think of prior as providing equivalent information to

- ν_0 prior observations
- with σ_0^2 average squared deviation from known μ

Estimating σ^2 of healing rates in population of newts

- Suppose μ was known to be 25.
- Suppose we had previously studied 2 newts and the average squared difference between their healing rate and 25 was 64.
- What is our appropriate prior?

- for newt data $\Sigma_{i=1}^{18}(y_i - 25)^2 = 1201$
- What is posterior

$$p(\sigma^2|y) \ \propto$$

- Recognize this as?

Noninformative prior for normal variance

- What inverse gamma prior would have information equivalent to 0 prior observations?

- How would you write this as a p.d.f for σ^2?

- Is it proper or improper?

- What characteristics would a dataset have to have in order to produce a proper posterior distribution for σ^2 if this prior were used?

Priors for normal *precision*

- if
$$\sigma^2 \sim IG(\alpha, \beta)$$
and
$$\tau^2 = \frac{1}{\sigma^2},$$
then
$$\tau^2 \sim G(\alpha, \beta)$$
- You must be careful of parameterizations of both gamma and inverse gamma distributions.