Wald's SPRT

George Woodworth

Hypotheses *(big is good)*

Inferior (H_0): $\theta \leq \theta_0 - \Delta$

Superior (H_2): $\theta \geq \theta_0$

Loss and Penalty Functions

Loss function	True status of the new intervention		
	Inferior	Neither	Superior
Decision	H_0: $\theta \leq \theta_0 - \Delta$	$\theta_0 - \Delta < \theta < \theta_0$	H_2: $\theta_0 < \theta$
Accept (acc)	**0**	**0**	**1**
Reject (rej)	**k**	**0**	**0**

Cost/penalty function: C(t) = C(R(t))

represents cost of data + penalty for delay

Wald SPRT

Sequential Likelihood Ratio Test

Observations: # of successes in n trials

$S_1, S_2, \ldots, S_n, \ldots$

Hypotheses: H_0 $p = p_0$, H_1 $p = p_1$

Likelihood Ratio:

$$LR_n = \exp\left(S_n \cdot \lambda + n \cdot \ln\left(\frac{1-p_1}{1-p_0}\right)\right), \text{ where } \lambda = \ln\left(\frac{p_1/(1-p_1)}{p_0/(1-p_0)}\right)$$

Note: λ is the log of the odds ratio

Decision Rule

If $LR_n < B$, then stop and accept H_0

If $LR_n > A$, then stop and reject H_0

Else take another observation.

Wald suggested

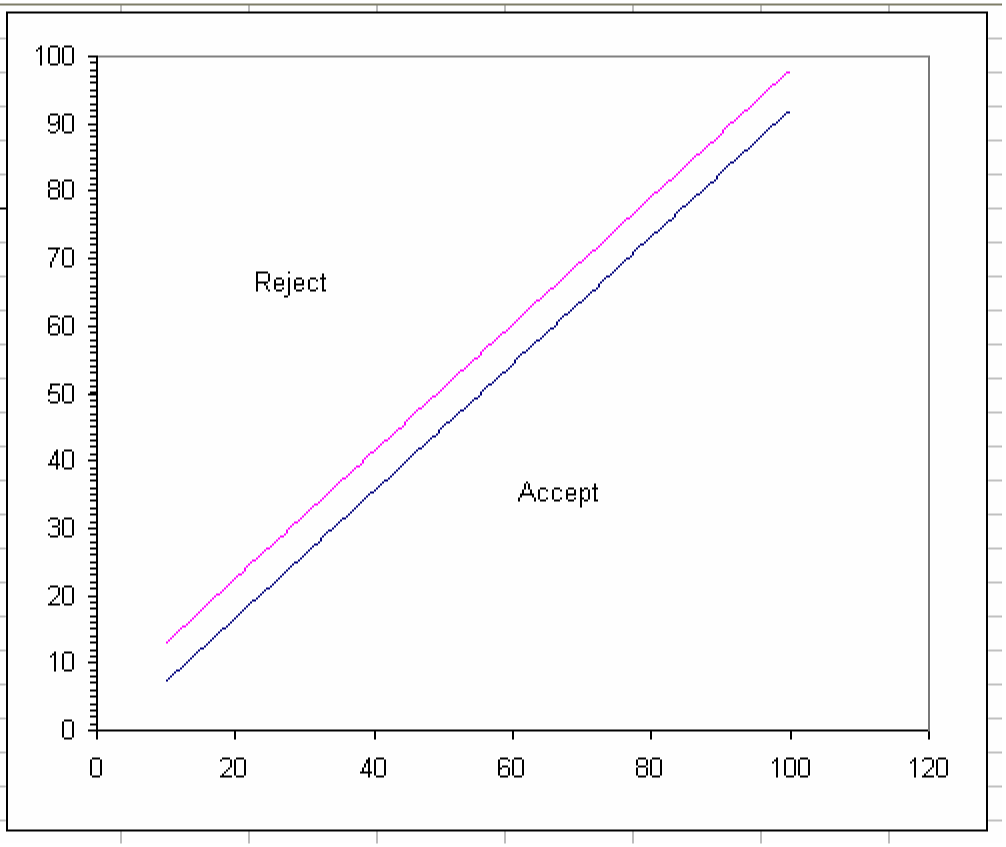
- $A = (1-\beta)/\alpha$
- $B = \beta/(1-\alpha)$

Stopping Boundaries for S_n

$$\frac{\ln(B) - nC}{\lambda} < S_n < \frac{\ln(A) - nC}{\lambda}, \quad \text{where C} = \ln\left(\frac{1 - p_1}{1 - p_0}\right)$$

p0	0.92	lambda	0.735707
p1	0.96	const	-0.69315
alpha	0.05	A	16
beta	0.2	B	0.210526

n	Lower	Upper
10	7.303626	13.19012
20	16.72514	22.61163
21	17.66729	23.55378
22	18.60944	24.49594
23	19.55159	25.43809
24	20.49375	26.38024
25	21.4359	27.32239
26	22.37805	28.26454
27	23.3202	29.20669
28	24.26235	30.14884
29	25.2045	31.091
30	26.14665	32.03315
31	27.08881	32.9753
32	28.03096	33.91745
33	28.97311	34.8596
34	29.91526	35.80175
35	30.85741	36.7439
36	31.79956	37.68606
37	32.74171	38.62821



Assignment

The excel file SPRT.xls is set up to compute lower and upper stopping boundaries for an SPRT with $p_0 = 0.92$ and $p_1 = 0.96$. For example at n=30, the LSB "accept" boundary is $S_n = 26$ and the USB "reject" boundary is $S_n = 33$.

Assuming a beta(1,1) prior, compute $P(p < .92 \mid Sn)$ and $P(p > .96 \mid Sn)$ for n = 20,40,60,80,100 and Sn=LSB and USB.