

Name: \_\_\_\_\_

1. (a) The following AR(2) model was fitted to a time series of size 30. Explain why the fitted model is stationary.

	ar1	ar2	intercept
Estimate	0.51	0.07	0.47
SE	0.18	0.18	0.29

- (b) Construct a 95% confidence interval for each of the three parameters. Which coefficient(s) are significant? Explain your answer to get credit.

- (c) Test the hypothesis that the stationary mean of the time series equals 1, at 5% significance level.

- (d) An AR(1) model was subsequently fitted to the data, resulting in the following fit:

	ar1	intercept
Estimate	0.55	0.48
SE	0.15	0.27

The AIC of the fitted AR(1) model equals 67.74, while that of the fitted AR(2) model equals 69.6. Which model do you prefer? Name two reasons in support of your preference.

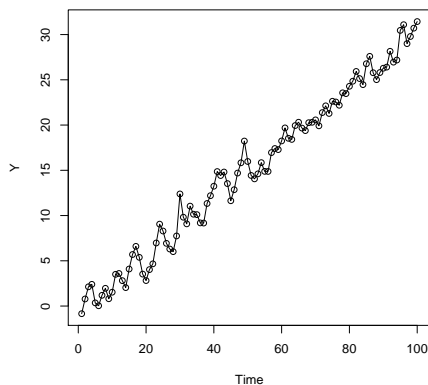
2. Let  $\{e_t\}$  and  $\{N_t\}$  be two independent time series, each of which is iid with zero mean and finite variances  $\sigma_e^2 > 0$  and  $\sigma_N^2 > 0$ .

(a) Define the process  $W_t = N_t + e_t - \theta_1 e_{t-1} - \theta_2 e_{t-2}$ , for all integer  $t$ , where  $\theta_1$  and  $\theta_2 \neq 0$  are two fixed parameters. Argue that  $\{W_t\}$  has finite memory of two lags.

(b) What ARIMA(p,d,q) model can be used to model  $\{W_t\}$ , i.e. determine the orders,  $p, d$  and  $q$ ?

(c) Let  $\{X_t\}$  be a stationary AR(2) process defined by the equation  $(1 - \phi_1 B - \phi_2 B^2)X_t = e_t$ , and  $Y_t = X_t + N_t$  where  $\{N_t\}$  and  $\{e_t\}$  are as in part (a). Compute  $(1 - \phi_1 B - \phi_2 B^2)Y_t$ . Hence, or otherwise, show that  $\{Y_t\}$  is an ARMA(2,2) process.

3. This question concerns whether or not the nonstationary time series shown in the right figure should be made stationary by taking the first difference. It is found that the first differences of the series can be fitted by an AR(8) model, based on AIC. A few ADF tests were carried out to assess the need for differencing the data, which are reported below.



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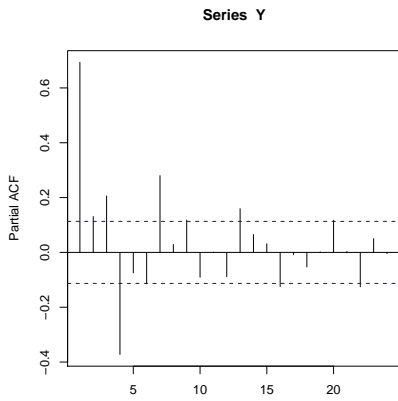
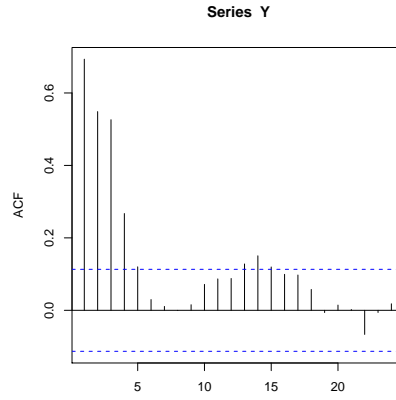
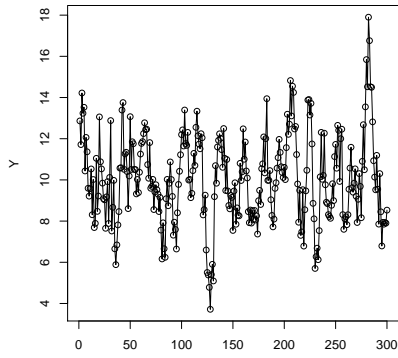
ADF.test(Y,selectlags=list(Pmax=0),itsd=c(0,0,0))
      Estimate Std. Error t value Pr(>|t|)
adf.reg   0.014      0.008   1.788   0.1
ADF.test(Y,selectlags=list(Pmax=0),itsd=c(1,0,0))
      Estimate Std. Error t value Pr(>|t|)
adf.reg  -0.009      0.015  -0.642   0.1
ADF.test(Y,selectlags=list(Pmax=0),itsd=c(1,1,0))
      Estimate Std. Error t value Pr(>|t|)
adf.reg  -0.528      0.09  -5.844  0.01
ADF.test(Y,selectlags=list(select.lag='aic',Pmax=8),itsd=c(1,1,0))
      Estimate Std. Error t value Pr(>|t|)
adf.reg  -0.843      0.108  -7.813  0.01

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Lag orders: 1 3 8

- (a) Explain the meaning of the option `itsd=c(1,1,0)`
- (b) Among the above 4 ADF tests, which one is the most appropriate test for checking the need for differencing? Explain your answer to get credits.
- (c) What is your recommendation on whether or not differencing is required? Explain your answer. If differencing is not recommended, what do you recommend to do to deal with the non-stationarity in the data.

4. Identify an ARIMA(p,d,q) model for the time-series variable  $Y$ , given its ACF, PACF and EACF. Explain your reasoning to get credits.



	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	x	x	x	x	x	o	o	o	o	o	o	o	x	x
1	x	x	x	o	o	o	o	o	o	o	o	o	o	o
2	x	x	x	x	o	o	o	o	o	o	o	o	o	o
3	x	x	x	x	o	o	o	o	o	o	o	o	o	o
4	x	x	x	o	x	o	o	o	o	o	o	o	o	o
5	x	o	x	o	o	o	x	o	o	o	o	o	o	o
6	x	x	x	x	o	x	o	o	o	o	o	o	o	o
7	x	x	x	o	o	x	x	o	x	o	o	o	o	o