May, 8, 2001. Final Exam for S156: Applied time series analysis

Name: ____________________

Below, $\{a_t\}$ denotes a sequence of iid random variables with zero mean and finite variance $\sigma_a^2 > 0$.

1. Let $Z_1, Z_2, \cdots, Z_{100}$ be an ARMA(1,1) process: $(1 - 0.5B)Z_t = (1 + 0.5B)a_t$. Listed below are the last two observed and fitted values:

	$t = 99$	$t = 100$
Z_t	3	1
$\hat{Z}_{t-1}(1)$	1	0

   (a) Predict Z_{101}, Z_{102}, and Z_{103}.

   (b) Find the variance of the predicton errors in part (a). (Assume the noise variance to be 1.)

   (c) Compute 95% prediction intervals for Z_{101}, Z_{102}, and Z_{103}.

2. From a series of length 100, we have computed $r_1 = 0.8, r_2 = 0.4, r_3 = 0.2$, and $\bar{Z} = 1$.

   (a) Assume that an AR(1) modelx: $Z_t = \phi_0 + \phi_1 Z_{t-1} + a_t$ is appropriate for this data set. Find consistent estimates for the parameters, using the method of moments. (No need to estimate the noise variance.)

   (b) Suppose that after examining the fitted AR(1) model, a statistician thought that an ARMA(1,1) model: $Z_t = \phi_0 + \phi_1 Z_{t-1} + a_t - \theta_1 a_{t-1}$ may be more appropriate. Find consistent estimates for ϕ_0 and ϕ_1.

3. This question concerns the empirical identification of ARIMA models.

   (a) The sample ACFs for a series and its first difference are given below (n=100):

				lag			
ACF	1	2	3	4	5	6	7
Z_t	0.55	0.58	0.53	0.53	0.52	0.53	0.43
ΔZ_t	-0.55	0.063	0.026	0.041	0.073	0.15	-0.18

   Based on this information, which ARIMA model(s) would you consider for the series. Explain your reasoning for credits.

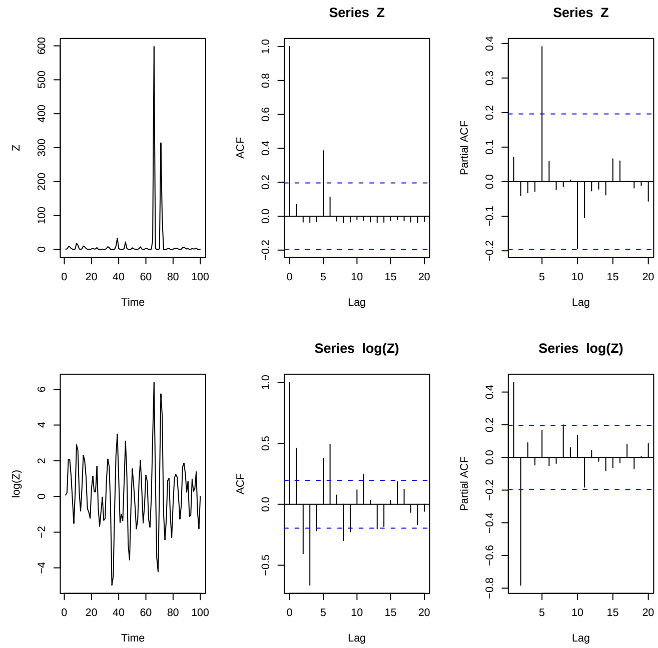

Figure 1:

(b) The time-series plot of 100 observations of a variable Z, and its acf and pacf plots are shown in Fig. 1. Also shown in the figure are similar plots for the log-transformed data. Based on this information, state whether the data should be analyzed on the original scale or the logarithmic scale, and which ARIMA model(s) would you consider for the series. Explain your answers for credits.

4. An AR(1) model was fitted to a time series with 100 observations.

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> fit1

Call:
arima0(x = Z, order = c(1, 0, 0))

Coefficients:
         ar1  intercept
      0.4788     0.0966

Approx standard errors:
         ar1  intercept
      0.0898     0.2495

sigma^2 estimated as 1.722:  log likelihood = -169.2,  aic = 342.41
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Figures 2 and 3 show some of the diagnostics check with the fitted AR(1) model. Comment on goodness of fit of the AR(1) model. In particular, comment on whether the residuals (i) have any outliers, (ii) conform to the iid normal assumption, and (iii) are of constant variance. If you judge the fitted AR(1) model inadequate, that is, the fit is poor, what model would you consider for the data, in view of the information from the residuals. You must explain your comments to get credits.

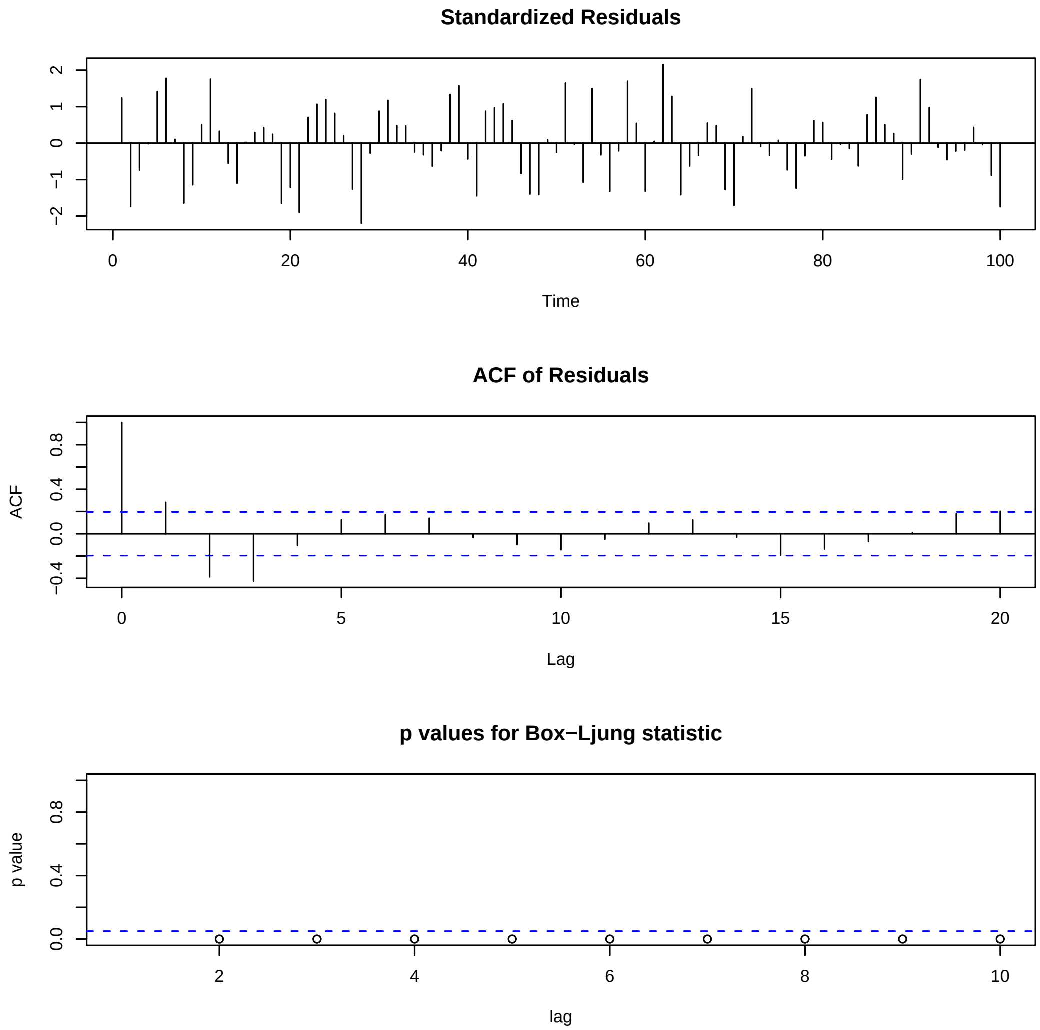

Figure 2: The upper figure shows the standardized residual plot, the middle figure shows the ACF of the residuals, and the bottom figure shows the p-values of the Box-Ljung statistics.

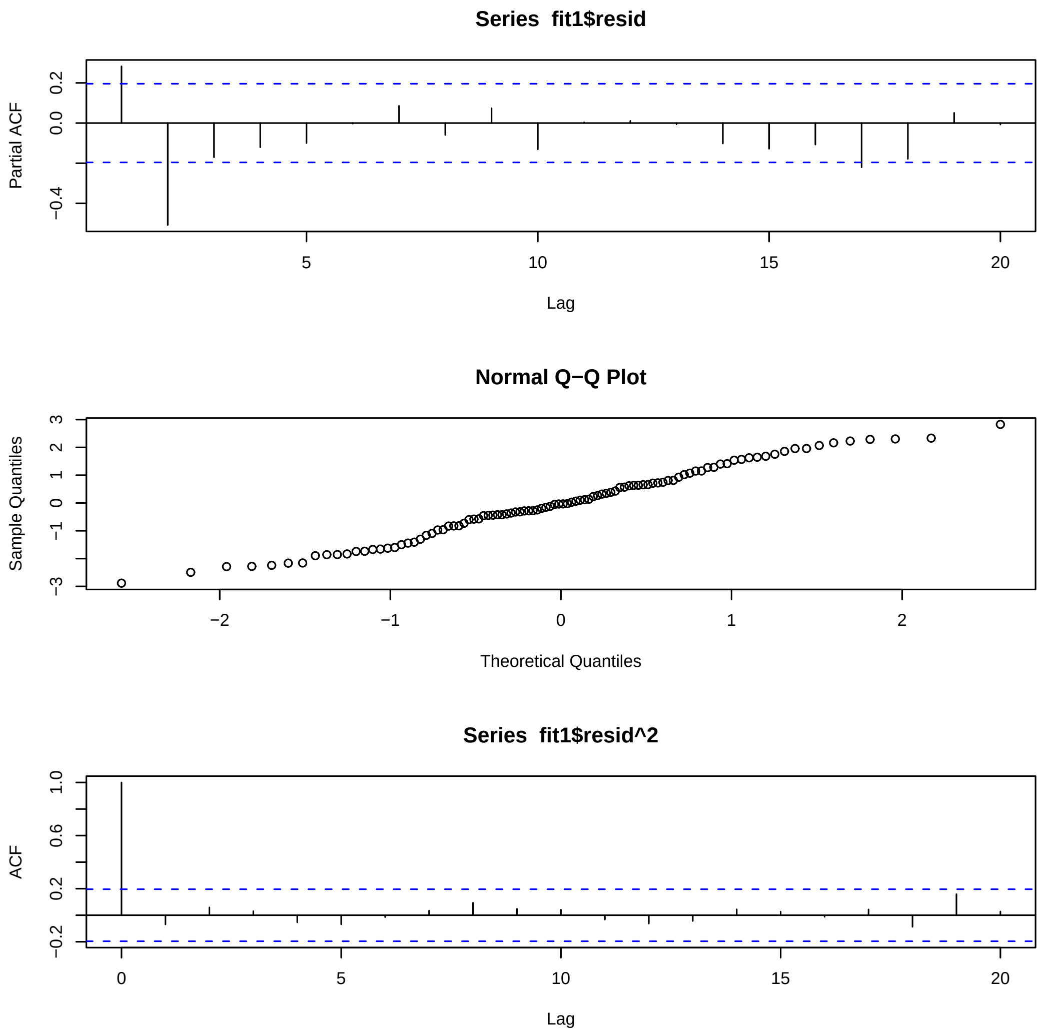

Figure 3: The upper figure shows the pacf of the residuals, the middle figure is the q-q normal plot of the residuals and the bottom figure shows the ACF of the squared residuals.

5. Consider the following model:

$$Z_t = \phi_0 + \phi_1 Z_{t-4} + a_t - \theta a_{t-1}.$$

(a) Determine the conditions on the parameters ϕ_0, ϕ_1 and θ under which the preceding model is stationary and invertible.

(b) Suppose that the conditions of invertibility holds and that we have data $Z_1, Z_2, \cdots, Z_n$. Show that for $\ell > 4$, the ℓ-step forecasts $\hat{Z}_n(\ell)$ satisfy the equation $\hat{Z}_n(\ell) = \phi_0 + \phi_1 \hat{Z}_n(\ell - 4)$.

(c) Consider the seasonal model with $\phi_0 = 0, \phi_1 = 0.8$ and $\theta = 0.8$. Suppose that the next five values are predicted to be $1, -3, 5, 6$ and 0.8, Construct an approximate 95% prediction interval for the 100-step ahead prediction, i.e., $\hat{Z}_n(100)$. (You may assume the noise variance to be one, and that this model is stationary.)