Final Examination for S156: Applied Time Series Analysis, Spring,94.

Name: ____

Instructions: This is a CLOSED book exam but you are allowed to have a sheet of paper with formulas, definitions,...,etc., written on both sides. Examination time is two hours. There are FOUR questions. Attempt all questions and write down your answers in the space provided below. Show all your workings. If you do not have enough space to write your answer, you can continue writing on the back of the papers and then indicate clearly that you have so done.

1.(32 points) State whether the following statements are True or False. You MUST show your reasoning in order to get credit.

(a)(8 points) For any invertible stationary ARMA model and any l, the forecasting variance of the l-step ahead predictor never exceeds the variance of the stationary distribution.

(b)(8 points) Let $\{Z_t\}$ satisfy the equation: $(1 - \phi B^4)Z_t = a_t$ where $|\phi|$ is less than 1 and $\{a_t\}$ is *iid* N(0,1). Then the theoretical acf for $\{Z_t\}$ is equal to zero except for lags that are multiples of four.

(c)(8 points) Let $Z_t = \alpha + \beta t + a_t$ where $\{a_t\}$ is *iid* N(0,1). Let $W_t = (1 - B)Z_t$. Then $\{W_t\}$ is a non-invertible MA(1) process.

(d)(8 points) An AR(1) model: $(1 - 0.9B)X_t = a_t$ is fitted to a data set. The one-step ahead forecast, $\hat{X}_N(1)$, is 0.5. The value of X_{N+1} is later observed to be 1. Without refitting the model, $\hat{X}_{N+1}(1)$ should be 0.8.

2.(25 points) Let $Z_1, Z_2, \cdots, Z_{100}$ be an SARIMA(0,0,1)×(1,0,0)4 process: $(1 - 0.9B^4)Z_t = (1 - .5B)a_t$ with $\{a_t\}$ being *iid* N(0,1). Listed below are the last five values of the Z's and the corresponding residuals:

t	96	97	98	99	100
Z_t	24	17	18	20	25
residuals	0	1	0	-1	1

(a)(8 points) Compute the forecasts for the next four Z's, i.e., Z_{101} to Z_{104}.

(b)(8 points) Determine the forecasting variance of the forecasts made in part (a).

(c)(9 points) It is later reported that $Z_{104} = 29$. Test whether this is an unusual observation. (You should set up the null hypothesis and derive an appropriate test statistic. No tables should be needed. State any assumptions made in your solution)

3.(20 points) Mr. Smith looks at a particular time series and concludes that it is well represented by the AR(2) model: $Z_t = 0.88Z_{t-1} + 0.1Z_{t-2} + a_t$. Mr. Jones looks at the same series and decides that an IMA(1,1) model radically different from that proposed by Mr. Smith is appropriate, namely, $Z_t = Z_{t-1} + a_t - 0.1a_{t-1}$.
(a)(8 points) Write down the AR(∞) representation of Mr. Jones' model. Determine the values of the first three π weights and give the general formula for the other π weights.

(b)(5 points) What does comparison of the result obtained in part (a) with Mr. Smith's model suggest about how radically different the two models really are?

(c)(7 points) How would you characterize the kind of long-run behavior implied by each of the models?

4.(23 points) The next three pages contain a record of a Minitab session done by Billy on the variable 'Z' stored in C1. Billy concluded that the IMA(1,1) model $(1-B)Z_t = (1-0.3209B)a_t$ provides an adequate fit for the data set. Do you agree with Billy's conclusion? Explain your answer. In the case that you disagree with Billy, what model would you propose to try in the next phase of the modeling? Explain your strategy. You should supply quantitative arguments in all the explanations.