Apr., 2, 2001. Exam 2 for S156: Applied time series analysis

Name: ____________________

Below, $\{a_t\}$ denotes a sequence of white noise with zero mean and finite variance $\sigma_a^2 = 1$.

1. For each of the following ARIMA(p,d,q) model, what are the values of p,d and q. Furthermore, state whether the model is stationary and/or invertible. Explain your answer briefly. (It suffices to verify the conditions for stationarity and invertibility for the models.)

   (a) $Z_t = 5 + a_t + 1.5a_{t-1} + .5a_{t-2}$

   (b) $Z_t - \frac{5}{6}Z_{t-1} + Z_{t-2}/6 = 1 + a_t + 2a_{t-1}$

   (c) $Z_t - 0.5Z_{t-1} - 0.5Z_{t-1} = a_t + 0.5a_{t-1}$

2. Consider the following stationary ARMA(2,1) process: $Z_t + Z_{t-1} + 0.6Z_{t-2} = a_t - 0.5a_{t-1}$.

   (a) Show that its autocorrelation function (acf) satisfies the equation: $\rho_k + \rho_{k-1} + 0.6\rho_{k-2} = 0, k \geq 2$.

   (b) Verify that the acf display a damped sine wave behavior. Find the frequence and period of the acf.

   (c) Given a realization of size 200 from the preceding ARMA(2,1) model, how many "cycles" do you expect to see in the series? Explain your answer.

3. Consider two models:

$$\begin{aligned} A:&\ (1 + 1.3B + 0.65B^2 + 0.325B^3 + 0.1625B^4 + 0.05B^5)Z_t = a_t; \\ B:&\ (1 + .8B)Z_t = (1 - .5B)a_t. \end{aligned}$$

(a) Find the π weights for the two models.

(b) Are these two models similar? Explain your answer.

4. Suppose that $(1-B)Z_t = -1. + (1-.5B)a_t$, where $Z_0 = 1$ and (a_t) is iid N(0,1).

(a) Show that $\mu_t - \mu_{t-1} = -1$ for $t \geq 1$ where $\mu_t = E(Z_t)$. Hence, or ortherwise, find the mean of Z_t, for $t \geq 0$. Does $\mu_t = E(Z_t)$ converge to a finite limit for t large?

(b) Consider another model: $(1-0.95B)Z_t = -1. + (1-.5B)a_t$, where $Z_0 = 1$ and (a_t) is iid N(0,1). Explain briefly why $\mu_t = E(Z_t)$ converges to a limit for t large? Find the limit.