Feb., 13 , 1995. Exam 1 for S156: Applied time series analysis

Name: _____

1. Let $X_t = 1 + a_t - .5a_{t-1}, t = 1, 2, 3, \cdots$, where (a_t) is iid N(0,2). That is, the variance of a_t is 2.
(a) Compute the mean of X_t.

(b) Compute the variance of X_t.

(c) Find the correlation between X_t and a_{t-1}.

2. Let (X_t) be a stationary AR(1) process: $X_t = 1 - .5X_{t-1} + a_t$ where (a_t) is iid N(0,σ_a^2).
(a) Find the mean of X_t. (Hint: write the model in the form of $X_t - \mu = \phi(X_{t-1} - \mu) + a_t$).

(b) If the variance of X_t is equal to 1, find σ_a^2.

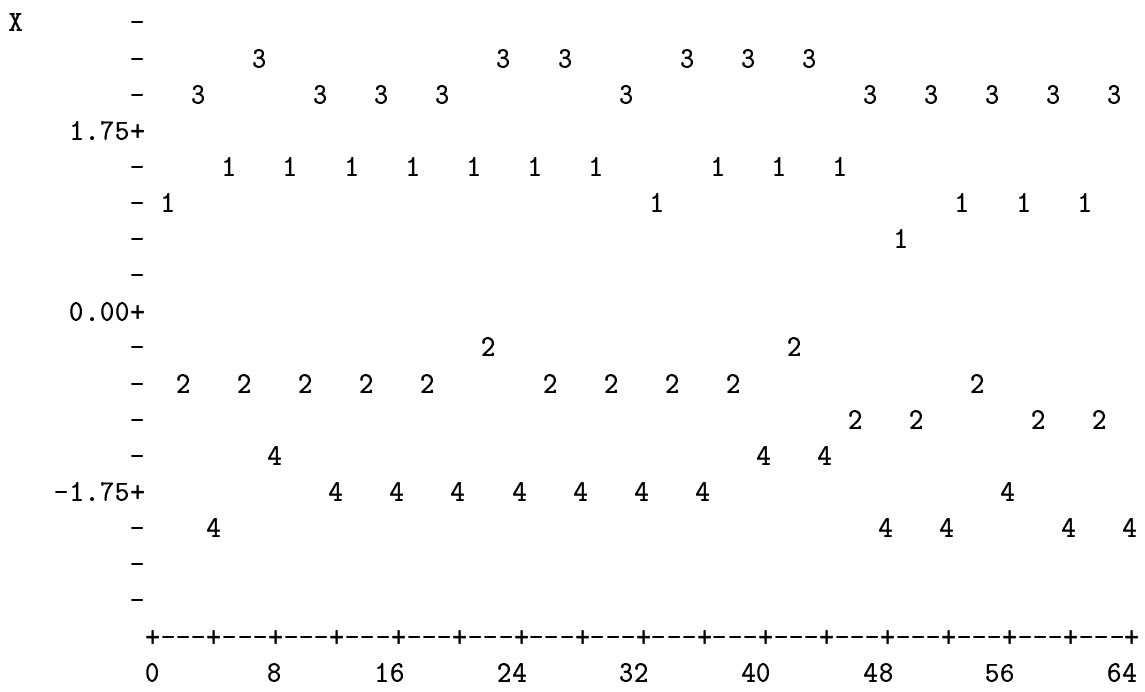
(c) Compute the ρ_1, ρ_2 and ρ_3, the lag 1 to lag 3 autocorrelations of (X_t).

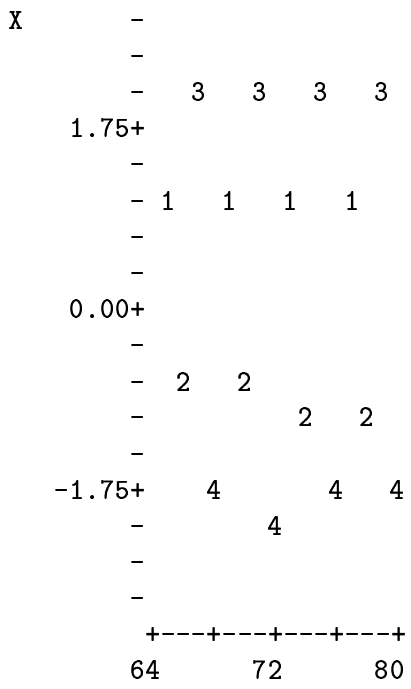
3. Let (X_t) be an MA(1) process: $X_t = a_t - \theta a_{t-1}$ where (a_t) is iid N(0,1). Let $Y_t = X_t + X_{t-1}, t = 1, 2, \cdots$.
(a) Show that (Y_t) is an MA(2) process.

(b) Suppose that $\theta = .4$. Compute the auto-correlation function of (Y_t).

4. The following pages record a Minitab session done by Peter on a time series which consists of 80 quarterly data on the variable 'X'. Peter fitted the following model: $X_t = \beta_1 Q1_t + \beta_2 Q2_t + \beta_3 Q3_t + \beta_4 Q4_t + e_t$. Here Qk is the dummy variable for the k-quarter. For example, $Q1_t$ is one if t is a first quarter and 0 otherwise. Peter concluded that e_t is independent and identically Normally distributed. Do you agree with his conclusion that (e_t) is a sequence of white noise? Explain your answer.

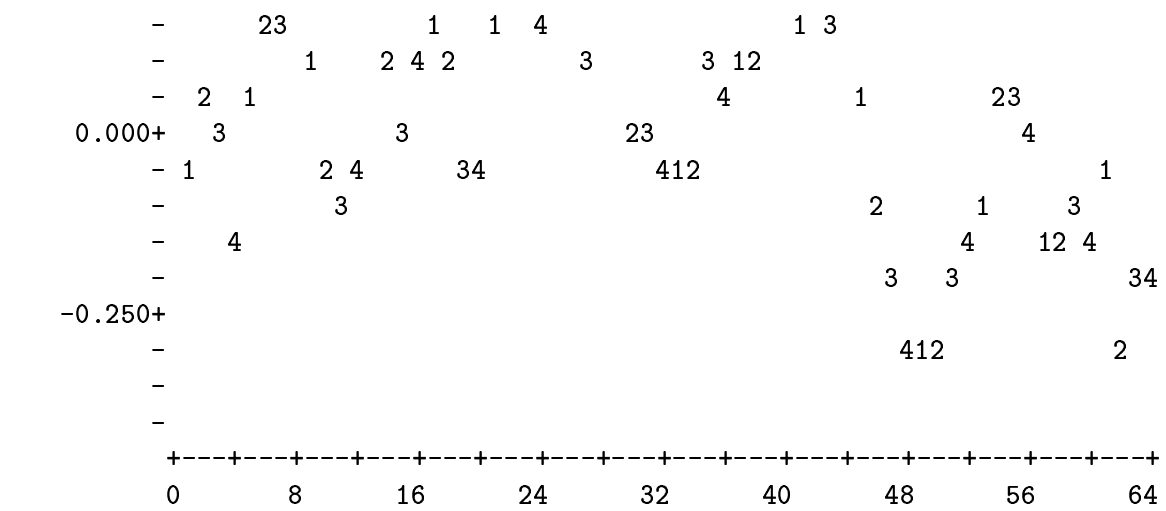
```
MTB > tsplot 4 c10
```

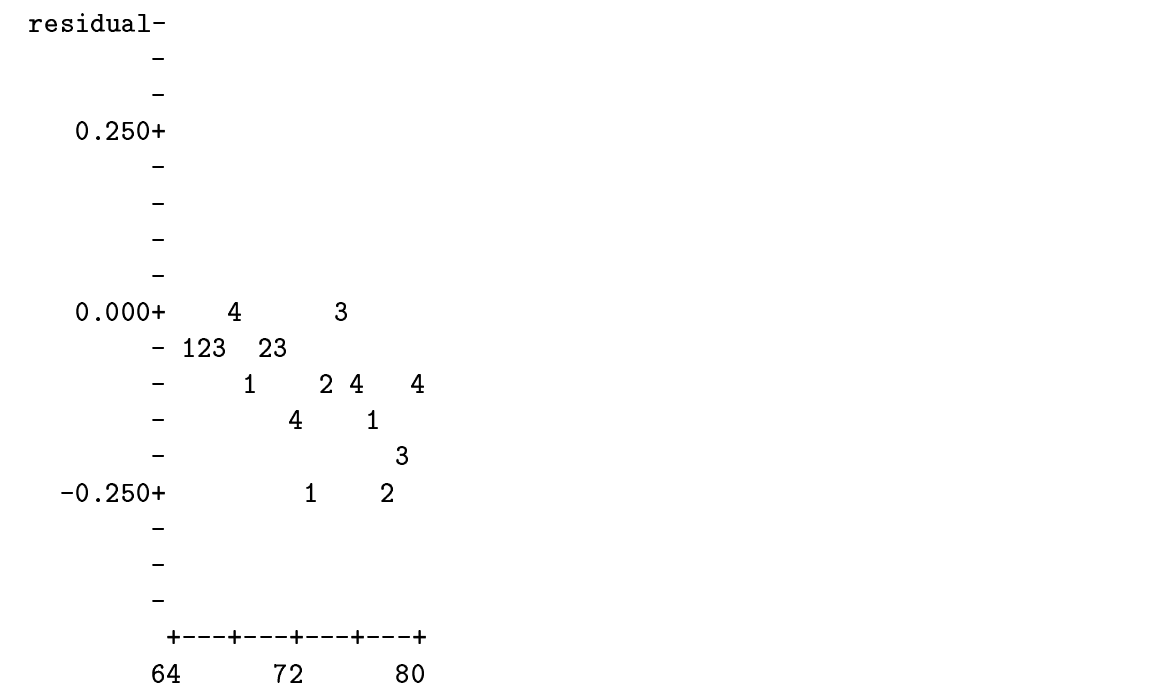





```
# Q1 is the dummy varibale for the first quarter; Q2 the dummy variable for the
# second quarter, etc.
```

```
MTB > regress 'X' 4 'Q1' 'Q2' 'Q3' 'Q4';
SUBC> noconstant;
SUBC> residuals in c20.

The regression equation is
X = 1.19 Q1 - 0.797 Q2 + 2.21 Q3 - 1.81 Q4

Predictor       Coef       Stdev    t-ratio        p
Noconstant
Q1           1.19043     0.03636      32.74    0.000
Q2          -0.79708     0.03636     -21.92    0.000
Q3           2.21107     0.03636      60.81    0.000
Q4          -1.80619     0.03636     -49.67    0.000

s = 0.1626
Analysis of Variance

SOURCE       DF          SS          MS         F        p
Regression    4     204.072      51.018   1929.36    0.000
Error        76       2.010       0.026
Total        80     206.082

1
SOURCE       DF      SEQ SS
Q1            1      28.343
Q2            1      12.707
Q3            1      97.777
Q4            1      65.246

Unusual Observations
Obs.       Q1          X        Fit  Stdev.Fit   Residual    St.Resid
 42      0.00    -0.4562    -0.7971     0.0364     0.3408       2.15R
 49      1.00     0.8655     1.1904     0.0364    -0.3250      -2.05R

R denotes an obs. with a large st. resid.

MTB > name c20 'residual'
MTB > tsplot 4 c20

residual-
        -                                          2
        -        4             23               3
   0.250+                          1  4           4   4
        -             1            2  1
```






```
1
 MTB > stem 'residual'

 Stem-and-leaf of residual  N  = 80
 Leaf Unit = 0.010


     3   -3 100
     4   -2 8
     9   -2 33110
```

```
  13   -1 8765
  19   -1 442221
  28   -0 999998755
 (16)  -0 4443333322221000
  36    0 01234
  31    0 56778888
  23    1 0001344
  16    1 555599
  10    2 233
   7    2 56899
   2    3 04


MTB > nscore c20 c21
MTB > name c21 'nscore'
MTB > # Normal score plot
MTB > plot c21 c20

 nscore  -                                                        *
         -
         -                                                    2
      1.6+                                                  ***
         -                                               3 *
         -                                          5 **
         -                                     222* *
         -                                *22*4
      0.0+                           622
         -                        *26*
         -                    242
         -                *222
         -              2**
     -1.6+          *  2
         -        **
         -
         -       *
           --------+---------+---------+---------+---------+--------residual
               -0.30     -0.15      0.00      0.15      0.30


MTB > runs 'residual'

    residual

    K =    -0.0000
```

```
    THE OBSERVED NO. OF RUNS =  13
    THE EXPECTED NO. OF RUNS =  40.6000
    36 OBSERVATIONS ABOVE K   44 BELOW
              THE TEST IS SIGNIFICANT AT  0.0000

MTB > acf c20

ACF of residual

          -1.0 -0.8 -0.6 -0.4 -0.2  0.0  0.2  0.4  0.6  0.8  1.0
            +----+----+----+----+----+----+----+----+----+----+
  1   0.768                          XXXXXXXXXXXXXXXXXXX
  2   0.560                          XXXXXXXXXXXXXX
  3   0.420                          XXXXXXXXXXX
  4   0.314                          XXXXXXXXX
  5   0.246                          XXXXXXX
  6   0.140                          XXXXX
  7   0.076                          XXX
  8   0.037                          XX
  9   0.081                          XXX
 10   0.136                          XXXX
 11   0.177                          XXXXX
 12   0.216                          XXXXXX
 13   0.279                          XXXXXXXX
 14   0.372                          XXXXXXXXX
 15   0.411                          XXXXXXXXXX
 16   0.399                          XXXXXXXXXX
 17   0.287                          XXXXXXXX
 18   0.140                          XXXX
```