

Feb., 25, 2002. Exam 1 for S156: Applied time series analysis

Name: _____

Below, $\{a_t\}$ denotes a sequence of iid random variables with zero mean and finite variance $\sigma_a^2 = 1$.

1. Let $(Z_t)_{t=0,1,2,\dots}$ be a stationary time series with zero mean, variance equal to 2 and *auto-correlation* function given by $\rho_0 = 1, \rho_1 = -0.5, \rho_k = 0, k \geq 2$.

(a) (3 points) Compute the *auto-covariance* function of $\{Z_t\}$.

(b) (3 points) Compute the mean of $Z_t - 0.5Z_{t-1}$.

(c) (3 points) Express the variance of $\bar{Z}_n = (Z_1 + Z_2 + \dots + Z_n)/n$ in terms of n and the auto-covariance function of $\{Z_t\}$.

2. State whether or not each of the following models for $\{Z_t\}$ is weakly (second-order) stationary. To get credits, you must explain your answer.

(a) (5 points) $Z_t = \prod_{j=1}^t a_j = a_1 \times a_2 \times \dots \times a_t$ for $t = 1, 2, \dots$.

(b) (5 points) $Z_t = a_t + ta_{t-1}$ for $t = 1, 2, \dots$.

3. State for each of the following model whether it is stationary and whether it is invertible. Explain your answers to get credit.

(a) (5 points)

$$Z_t = 1 + 0.5Z_{t-1} + a_t, t = 1, 2, \dots$$

(b) (5 points)

$$Z_t = a_t - 1.5a_{t-1} + 0.5a_{t-2}, t = 1, 2, \dots$$

4. Consider the difference equation

$$Y_t = 0.6Y_{t-1} - 0.25Y_{t-2}, t = 2, 3, \dots.$$

(a) (4 points) Show that all solutions of this difference equation are damped sinusoidals and hence tend to zero as $t \rightarrow \infty$.

(b) (3 points) Find the quasi-period of the solutions.

(c) (3 points) Suppose we modify the preceding difference equation to

$$Y_t = 0.6Y_{t-1} - 0.25Y_{t-2} + 3, t = 2, 3, \dots.$$

Find the limit of the solutions of the modified equation, as $t \rightarrow \infty$.