Chapter 10: Control Charts for Continuous Variables

Common and Special Causes of Variation

Statistical Control

Standard Deviation and Mean Control Charts

Common and Special Causes of Variation (page 276)

In a constant-cause system the variations in a measured variable are considered due to chance and to remain in the system unless the process is itself altered.

Such causes are referred to as

***common causes*.**

The variation observed is considered to be the effect of many, individually small, unobserved influences.

Examples of *Common Causes*:

- Variations in environmental variables such as temperature and humidity
- Variations among machines
- Variations among worker's skills
- Variations in raw materials

Processes are also subject to the influence of ***special causes***. (page 276)

Special causes are individually important and affect process results only some of the time. They arise because of special circumstances.

Because of their underlying differences, dealing with the two kinds of causes requires different techniques.

A process is considered to be ***in control*** (or ***stable***) if only common causes are operable and no special causes are influencing the variability of process results.

Examples of
Special Causes:

- Temporary change to a new supplier
- Temporary worker not properly trained
- Intermittent power failure
- Ice storm
- Bird in the toothpaste vat

Overreacting to common-cause variation usually leads to “tampering” with a process that may already be in control.

☆ This in turn leads to ***increased*** variation in process results.

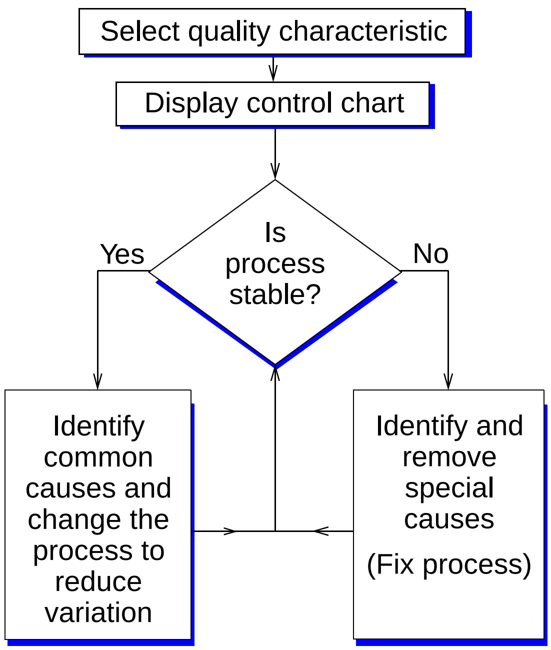
☆ On the other hand, failing to detect and deal with special causes means a lost opportunity to eliminate variation in the process.

✪ Fortunately, control charts, invented by Dr. Walter Shewhart in 1924, display information about a process that permits the differentiation between common and special causes of variation.

What is Statistical Control?

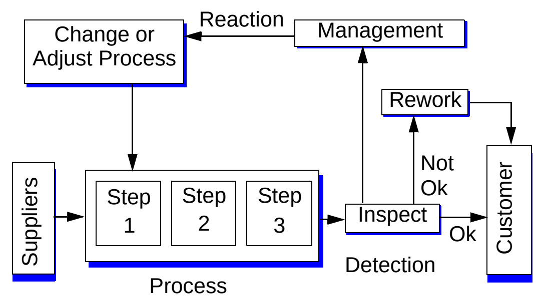
In 1944 Dr. W. Edwards Deming wrote (page 275)

“There is no such thing as constancy in real life. There is, however, such a thing as a constant-cause system. The results produced by a constant-cause system vary, and, in fact, may vary over a wide band or a narrow band. They vary, but they exhibit an important feature called stability, Why apply the terms constant and stability to a cause system that produces results that vary? Because the same percentage of these varying results continues to fall between any pair of limits hour after hour, day after day, so long as the constant-cause system continues to operate. It is the distribution of results that is constant or stable. When a manufacturing process behaves like a constant-cause system, producing inspection results that exhibit stability, it is said to be in statistical control. The control chart will tell you whether your process is in statistical control.”



The Old Way:

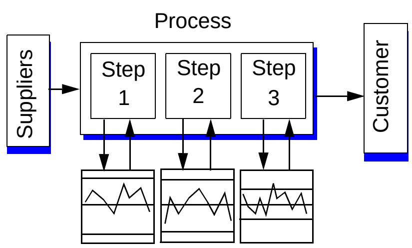
Detection-Reaction Systems



Action on the output is *past-oriented* because it involves *detecting* non-conforming output that *has already been produced*.

The New Way:

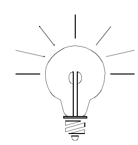
Process Control Systems



Monitor each step of the process with control charts and adjust as needed.

(Also get suppliers to use control charts.)

Important!



- ***specification limits*** are the voice of the customer
- ***control limits*** are the voice of the process

Mean Charts (Xbar Charts) for Process Control

(page 276)

To study variation of process results over time we must look at sequence plots of process data.

In order to assess both the mean level and variability of the process over time a sample of data values taken reasonably close in time is required.

Rational Subgroups

The aim of rational subgrouping is to include only common cause variation within subgroups with all special causes occurring between subgroups.

For example, to control the thickness of processed meat, such as bologna, it is common practice to cut and measure the thickness of five slices from one "log" of meat each hour.

In some processes it is possible to measure the quality characteristic of interest on a sample of n items at regular time points.

Each of these samples is called a ***subgroup***.

The subgroups may be selected from all of the product produced in the last hour for example or from one shift.

In other processes it may be necessary to take the n measurements over a day or week to produce the subgroup.

The sampling frequency will depend on the process being studied and, in particular, on the frequency at which it is feared special causes may affect the process.

We will let $\bar{y}_i$ and s_i denote the mean and standard deviation, respectively, of the i^{th} subgroup and suppose that k subgroups are available for analysis.

The ***Central Limit Effect*** says that the distribution of $\bar{y}_i$ will be approximately normal when several measurements from a stable process are averaged.

Thus if a constant-cause system is in effect, then we know what to expect from $\bar{y}_i$.

Unusual behavior in $\bar{y}_i$ leads us to question whether a constant-cause system is operating.

A graphical display of the sequence of $\bar{y}_i$'s for i = 1, 2,..., k (the control chart) helps us look for unusual values.

We begin with the data shown next. Twenty-five subgroups each of size 5 are shown in the rows. The means and standard deviations are also given.

Subgroup Number	Samples					Mean	St. Dev.
	1	2	3	4	5		
1	101.5	98.5	97.0	102.3	99.4	99.74	2.17
2	101.1	100.2	100.4	97.0	101.8	100.10	1.84
3	98.8	99.9	98.2	101.4	99.1	99.48	1.24
4	100.4	99.8	99.4	99.1	97.3	99.20	1.17
5	99.2	101.7	101.6	100.0	100.5	100.60	1.07
6	96.8	101.9	98.0	102.3	100.0	99.80	2.39
7	102.9	98.1	102.3	100.1	99.9	100.66	1.95
8	97.5	100.1	101.9	95.5	101.1	99.22	2.66
9	98.3	98.4	96.3	98.8	100.2	98.40	1.40
10	98.5	97.0	100.6	103.2	102.7	100.40	2.66
11	100.8	98.2	101.3	102.1	101.3	100.74	1.49
12	103.2	101.0	97.6	100.1	100.8	100.54	2.01
13	99.5	100.1	101.2	100.2	99.6	100.12	0.68
14	100.2	94.9	99.4	103.7	103.0	100.24	3.49
15	97.3	101.8	99.2	101.0	100.7	100.00	1.78
16	100.9	99.6	102.9	100.8	99.4	100.72	1.40
17	99.8	97.9	100.7	100.3	99.3	99.60	1.09
18	99.9	99.3	100.6	101.1	103.3	100.84	1.54
19	96.1	101.1	104.1	97.4	102.1	100.16	3.33
20	98.3	99.2	100.7	98.2	100.9	99.46	1.29
21	98.4	104.7	100.0	98.2	99.2	100.10	2.67
22	101.9	97.8	98.1	103.4	99.0	100.04	2.48
23	101.7	96.8	100.9	100.8	101.8	100.40	2.06
24	101.8	102.9	102.9	98.8	101.5	101.58	1.68
25	102.3	100.9	100.1	99.4	101.1	100.76	1.09

Making A Mean Chart

(page 277)

- Find the mean, $\bar{y}_i$, for each subgroup
- Find the grand mean, $\bar{\bar{y}}$, by averaging the subgroup means

$$\bar{\bar{y}} = \frac{\bar{y}_1 + \bar{y}_2 + \cdots + \bar{y}_k}{k}$$

- Find the subgroup standard deviations, s_i
- Find the average standard deviation, $\bar{s}$

$$\bar{s} = \frac{s_1 + s_2 + \cdots + s_k}{k}$$

We estimate the 3 sigma control limits,

$$\mu \pm 3\frac{\sigma}{\sqrt{n}}$$

using $\bar{\bar{y}}$ to estimate μ and (essentially) $\bar{s}$ to estimate σ.

In fact, theory shows that $\bar{s}$ tends to underestimate σ so that there is an additional "fudge factor" thrown in. We combine the 3, the $\sqrt{n}$, and the additional fudge factor into a tabled constant we call simply a. These are tabled in Exhibit 10.3D, page 280.

Then we calculate

Upper Control Limit (UCL) = $\bar{\bar{y}} + a\bar{s}$

and

Lower Control Limit (LCL) = $\bar{\bar{y}} - a\bar{s}$

We also draw a Center Line (CL) at $\bar{\bar{y}}$.

Example

For our data $\bar{\bar{y}}$ = 100.12 and $\bar{s}$ = 1.86.

From the table with n = 5 we find a = 1.43

so that

UCL = $\bar{\bar{y}} + a\bar{s}$ = 100.12 + (1.43)1.86 = 102.78

and

LCL = $\bar{\bar{y}} - a\bar{s}$ = 100.12 – (1.43)1.86 = 97.46

with CL at 100.12.

As long as the sequence of subgroup means remains within the control limits there is no evidence that special causes are influencing the process and we say that the process is in control.

Anytime the sequence of subgroup means goes outside the control limits, we say that the process is out of control and treat that as strong evidence that special causes are influencing the process.

A process could go out of control at an isolated subgroup or it could continue out of control for several subgroups.

When the control chart signals that the process is out of control we need to find out the nature of the special cause and fix the process.

Making A Standard Deviation Chart
(page 284)

- Find the sequence of subgroup standard deviations, s_i
- Find the average standard deviation, $\bar{s}$

$$\bar{s} = \frac{s_1 + s_2 + \cdots + s_k}{k}$$

- Using the table in Exhibit 10.5A, page 285, calculate the upper and lower control limits on s_i from

  LCL = $b\bar{s}$

  and

  UCL = $c\bar{s}$

  Use $\bar{s}$ for the center line.

- Construct the control chart as a sequence plot of s_i with the control limits and center line displayed.

As long as the sequence of subgroup standard deviations remains within the control limits there is no evidence that special causes are influencing the process variability and we say that the process is in control with respect to variability.

Anytime the sequence goes outside the control limits, we say that the process is out of control with respect to variability.

Example

From our data, n = 5 and the factors from Exhibit 10.5A are b = 0.160 and c = 2.286.

So that the control limits are

LCL = $b\bar{s}$ = 0.160(1.86) = 0.298

and

UCL = $c\bar{s}$ = 2.286(1.86) = 4.25.

As long as our subgroup standard deviations remain within these control limits we should not tamper with the process. Variability is in control.

Process Analysis (page 288)

When first analyzing a processes we don't know what the situation is with respect to control. Frequently we are doing charts because we suspect we have special causes in our system.

Special causes will affect the calculation of "good" control limits so we may call them **trial control limits.**

For future process monitoring the trial control limits are subject to adjustment once we find and eliminate special causes.

Since mean charts are impossible to interpret in the absence of stable variability we always begin with standard deviation charts.

Examples of Control Charts in Business Applications

Automotive Washer Thicknesses

Ignition Key Measurements

Ice Cream Fill Amounts

Accounting Processes