

Key.

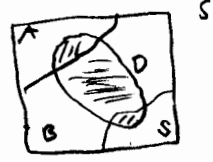
Quiz 7
Statistics for Business (22S:008, Bognar)

October 11, 2006

1. (5 pts) A large warehouse contains thousands of identically sized bolts. It is known that 25% of the bolts are made of aluminum (A), 70% are made of brass (B), and 5% are made of steel (S). Past experience has shown that 4% of the aluminum bolts, 6% of the brass bolts, and 2% of the steel bolts are defective (D). If a bolt is chosen at random from the warehouse, find the probability that the bolt is defective. *Hint: it may be helpful to draw the partitioned sample space S overlain with event D. Show all of your work using good notation.*

$$\begin{aligned}
 P(A) &= 0.25 \\
 P(B) &= 0.70 \\
 P(S) &= 0.05 \\
 P(D|A) &= 0.04 \\
 P(D|B) &= 0.06 \\
 P(D|S) &= 0.02
 \end{aligned}$$

$$\begin{aligned}
 P(D) &= P(A \text{ and } D) + P(B \text{ and } D) + P(S \text{ and } D) \\
 &= P(D|A)P(A) + P(D|B)P(B) + P(D|S)P(S) \\
 &= 0.04 \times 0.25 + 0.06 \times 0.70 + 0.02 \times 0.05 \\
 &= 0.053
 \end{aligned}$$



2. (8 pts) It is known that 27% of elderly adults suffer from cataracts. Suppose 9 elderly adults are randomly selected; let the random variable X equal the number with cataracts.

(a) (4 pts) What is the distribution of X? (Be sure to state all parameters.)

$$X \sim \text{Bin}(n=9, p=0.27)$$

(b) (4 pts) Find the probability that exactly 4 of 9 randomly selected elderly adults suffer from cataracts (i.e. find $P(X=4)$). Show all of your work using good notation.

$$P(X=4) = \binom{n}{x} p^x (1-p)^{n-x} = \binom{9}{4} 0.27^4 (1-0.27)^{9-4} = 0.1388$$

$$\binom{9}{4} = \frac{9!}{4!(9-4)!} = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2 \cdot 1} = 126$$

3. (7 pts) It is known that systolic blood pressures of executives follow a normal distribution with mean μ and standard deviation $\sigma = 15$. Suppose the mean systolic blood pressure of 72 randomly selected executives was $\bar{x} = 130$. Find an 80% CI for μ . Show all of your work using good notation.

$$\begin{aligned}
 X &\sim N(\mu, \sigma=15) \\
 n &= 72 \quad \bar{x} = 130
 \end{aligned}$$

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}} = 130 \pm 1.28 \frac{15}{\sqrt{72}} = 130 \pm 2.26 = (127.74, 132.26)$$

$$z_{.10} = 1.28$$