

Key.

Quiz 5
Statistics for Business (22S:008, Bognar)

September 27, 2006

1. (20 pts) Suppose a bowl has 5 chips; two chips are labeled "2", and three chips are labeled "3". Suppose two chips are selected at random without replacement. Let the random variable X equal the product of the two draws.

(a) (10 pts) Find the probability distribution of X . Show all of your work using good notation.

X	$P(X=x)$
4	$\frac{2}{20}$
6	$\frac{12}{20}$
9	$\frac{6}{20}$

$$P(X=4) = P(2_1 \text{ and } 2_2) = P(2_1)P(2_2|2_1) = \frac{2}{5} \cdot \frac{1}{4} = \frac{2}{20}$$

$$P(X=6) = P[(2_1 \text{ and } 3_2) \text{ or } (3_1 \text{ and } 2_2)] \stackrel{\text{m.c.}}{=} P(2_1 \text{ and } 3_2) + P(3_1 \text{ and } 2_2)$$

$$= P(2_1)P(3_2|2_1) + P(3_1)P(2_2|3_1) = \frac{2}{5} \cdot \frac{3}{4} + \frac{3}{5} \cdot \frac{2}{4} = \frac{12}{20}$$

$$P(X=9) = P(3_1 \text{ and } 3_2) = P(3_1)P(3_2|3_1) = \frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20}$$

(b) (3 pts) Find the probability that the product of the two draws is less than or equal to 6 (i.e. find $P(X \leq 6)$). Show all of your work using good notation.

$$P(X \leq 6) = P(X=6) + P(X=4) = \frac{14}{20}$$

(c) (7 pts) Find the probability that the product of the two draws is greater than 4 given that the product is less than or equal to 6 (i.e. find $P(X > 4 | X \leq 6)$). Show all of your work using good notation.

$$P(X > 4 | X \leq 6) = \frac{P[(X > 4) \text{ and } (X \leq 6)]}{P(X \leq 6)} = \frac{P(X=6)}{P(X \leq 6)} = \frac{12/20}{14/20} = \frac{12}{14} = \frac{6}{7}$$