Key.

Quiz 9

Statistics for Business (22S:008, Bognar) **March 28, 2007**

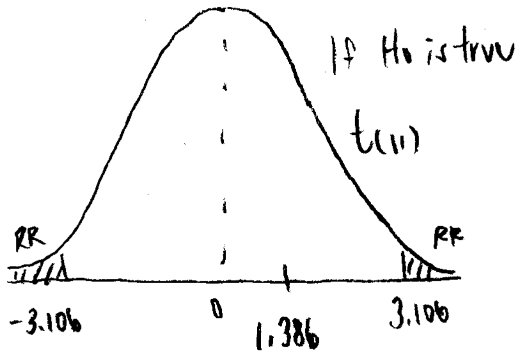
1. (*12 pts*) It is known that the tensile strength of a certain type of *aluminum* wire follows a normal distribution with mean μ pounds and standard deviation σ pounds. Suppose a researcher randomly selected 12 pieces of aluminum wire, and computed the sample mean $\bar{x} = 41.4$ and sample standard deviation $s = 3.5$.

   (a) (*7 pts*) Test $H_0 : \mu = 40$ versus $H_a : \mu \neq 40$ at the $\alpha = 0.01$ significance level. *You must determine the test statistic and critical value, plot the rejection region (be sure to label your graph), and state your decision and final conclusion. Show all of your work using good mathematical notation.*

1) $t^* = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{41.4-40}{3.5/\sqrt{12}} = \dfrac{1.4}{1.010} = 1.386$

2) $t_{\alpha/2,\,n-1} = t_{.005,11} = 3.106$

3)



Do not reject H0.
No evidence that $\mu \neq 40$

   (b) (*5 pts*) Approximate the p–value for the test in part (a). *Show all of your work using good mathematical notation.*

$p\text{-value} = 2P(t_{(11)} > 1.386) \in (0.10, 0.20)$ → Note: Dont reject H since p-value > ...

2. (*8 pts*) Suppose we wish to test $H_0 : \mu = 10$ versus $H_a : \mu \neq 10$ at the $\alpha = 0.05$ significance level. If the sample size $n = 9$, the sample mean $\bar{x} = 9.3$, and the p–value for the test is 0.30, find a 95% confidence interval for μ. Assume the population is normal with mean μ and standard deviation σ. *Assume σ is* not *known. Hint: first find the sample standard deviation s.*

$\bar{x} \pm t_{\alpha/2,\,n-1}\, s/\sqrt{n} = 9.3 \pm 2.306\, \dfrac{1.895}{\sqrt{9}}$

$t_{.025,8} = 2.306$

$= 9.3 \pm 1.457$

$= (7.843, 10.757)$

$p\text{-value} = 0.30 = 2P(t_{(8)} > |t^*|)$

$\Rightarrow 0.15 = P(t_{(8)} > |t^*|)$

$\Rightarrow |t^*| = 1.108$

$\Rightarrow t^* = -1.108$ since $\bar{x} <$ ...

$\Rightarrow -1.108 = \dfrac{\bar{x}-\mu}{s/\sqrt{n}} = \dfrac{9.3-10}{s/\sqrt{9}}$

$\Rightarrow s = \dfrac{9.3-10}{-1.108} \times \sqrt{9} = 1.8$...