

Key.

Quiz 12
Statistics for Business (22S:008, Bognar)

November 15, 2006

1. (20 pts) Consider the following contingency table which summarizes the incidence of tuberculosis (TB) for needle-sharing drug users versus the incidence of TB for non-needle-sharing drug users.

	TB	No TB	
Share	30 (24)	70 (76)	100
No Share	30 (36)	120 (114)	150
	60	190	250

Our goal is to test H_0 : needle sharing and TB are independent versus H_a : needle sharing and TB are not independent at the $\alpha = 0.10$ significance level.

- (a) (3 pts) Given that a randomly selected person shares needles, find the probability that he/she has TB. Also, find the probability a randomly selected non-needle-sharer has TB.

$$P(\text{TB} | \text{Share}) = 30/100 = 0.30$$

$$P(\text{TB} | \text{no Share}) = 30/150 = 0.20$$

- (b) (3 pts) Determine the expected count for each cell in the table. Show all of your work using good notation.

$$e_{11} = \frac{100 \times 60}{250} = 24$$

$$e_{12} = \frac{100 \times 190}{250} = 76$$

$$e_{21} = \frac{150 \times 60}{250} = 36$$

$$e_{22} = \frac{150 \times 190}{250} = 114$$

- (c) (3 pts) The expected counts are computed under what assumption?

That TB and needle sharing are independent

- (d) (4 pts) Compute the test statistic. Show all of your work using good notation.

$$\begin{aligned} \chi^2 &= \sum_{i=1}^{r=2} \sum_{j=1}^{c=2} \frac{(o_{ij} - e_{ij})^2}{e_{ij}} \\ &= \frac{(30-24)^2}{24} + \frac{(70-76)^2}{76} + \frac{(30-36)^2}{36} + \frac{(120-114)^2}{114} \\ &= 3.289 \end{aligned}$$

- (e) (4 pts) Approximate the p-value of the test. Show all of your work using good notation.

$$p\text{-value} = P(\chi^2_{(1)} > 3.289) \in (0.05, 0.10)$$

$$\hookrightarrow df = (r-1)(c-1) = (2-1)(2-1) = 1$$

- (f) (3 pts) Is there evidence that needle-sharers have a higher incidence of TB? Why?

Yes, since $p\text{-value} < \alpha$.