Key.

Quiz 10
Statistics for Business (22S:008, Bognar) **November 1, 2006**

1. (*20 pts*) The length of time *Brand X* batteries last in a hand–held game follow a normal distribution with mean μ_1 and standard deviation σ_1, while the longevity of *Brand Y* batteries follow a $N(\mu_2, \sigma_2)$ distribution. A child prodigy wanted to determine which brand lasted longer, so she decided to perform an experiment. She found that the mean longevity of 6 *Brand X* batteries was $\bar{x}_1 = 11.0$ hours with standard deviation $s_1 = 1.6$ hours, while the mean longevity of 7 *Brand Y* batteries was $\bar{x}_2 = 14.5$ hours with standard deviation $s_2 = 2.8$ hours. Since s_1 and s_2 are quite different, we assume that $\sigma_1 \neq \sigma_2$.

   (a) (*7 pts*) Find a 95% confidence interval for $\mu_1 - \mu_2$. *Show all of your work using good notation.*

$X \sim N(\mu_1, \sigma_1)$
$Y \sim N(\mu_2, \sigma_2)$
$n_1 = 6 \quad n_2 = 7$
$\bar{x}_1 = 11.0 \quad \bar{x}_2 = 14.5$
$s_1 = 1.6 \quad s_2 = 2.8$

$$\bar{x}_1 - \bar{x}_2 \pm t_{\alpha/2,\nu}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = 11.0 - 14.5 \pm 2.262\sqrt{\frac{1.6^2}{6} + \frac{2.8^2}{7}}$$

$$\downarrow$$

$$t_{.025,9} = 2.262$$

$$= -3.5 \pm 2.813$$

$$= (-6.313, -0.687)$$

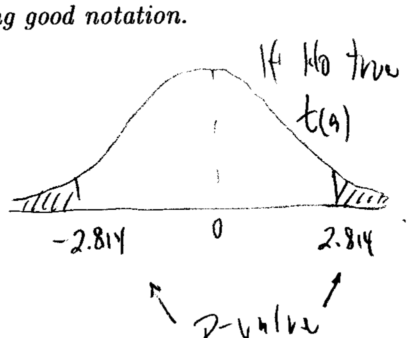
$$\nu = \frac{\left(s_1^2/n_1 + s_2^2/n_2\right)^2}{\frac{\left(\frac{s_1^2}{n_1}\right)^2}{n_1 - 1} + \frac{\left(\frac{s_2^2}{n_2}\right)^2}{n_2 - 1}} = \frac{\left(1.6^2/6 + 2.8^2/7\right)^2}{\frac{\left(\frac{1.6^2}{6}\right)^2}{6-1} + \frac{\left(\frac{2.8^2}{7}\right)^2}{7-1}} = \frac{2.39218}{0.24548} = 9.745 \downarrow 9$$

   (b) (*3 pts*) Based upon your answer in (1a), is there a significant difference in mean longevity between the brands? Why?

Yes, since CI lies entirely below 0 (i.e. since CI excludes 0).

   (c) (*7 pts*) Suppose we wish to test $H_0 : \mu_1 = \mu_2$ versus $H_a : \mu_1 \neq \mu_2$ at the $\alpha = 0.05$ significance level. Approximate the p–value of this test. *Show all of your work using good notation.*

$$t^* = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(11.0 - 14.5) - (0)}{\sqrt{\frac{1.6^2}{6} + \frac{2.8^2}{7}}} = -2.814$$



$$p\text{-value} = 2P(t_{(9)} > 2.814) \in (0.02, 0.04)$$

   (d) (*3 pts*) Based upon your answer in (1c), is there a significant difference in mean longevity between the brands? Why?

Yes, since p-value $< \alpha$ (i.e. we reject H_0).