

Generalized Levinson-Durbin Sequences, Binomial Coefficients and Autoregressive Estimation

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Abstract:

For a discrete time, second-order stationary process, the Levinson-Durbin recursion is used to determine the coefficients of the best linear predictor of the observation at time $k+1$, given data at times $1, \dots, k$, best in the sense of minimizing the mean square error. The double sequence of coefficients as k varies determines a Levinson-Durbin sequence. A generalized Levinson-Durbin sequence is also considered. Binomial coefficients form a generalized Levinson-Durbin sequence, and all generalized Levinson-Durbin sequences are shown to obey some summation formulas which generalize summations satisfied by binomial coefficients. The summation formulas are expressed in terms of the partial correlation sequence of the process. Levinson-Durbin sequences arise in the construction of autoregressive model coefficient estimates for the Yule-Walker, Burg and Kay estimators. The least squares autoregressive estimator, though, does not give rise to a Levinson-Durbin sequence. However, least squares fixed point processes, which yield least squares estimates of the coefficients unbiased to order $1/T$, where T is the sample length, can be combined to construct Levinson-Durbin sequences. By contrast, analogous Yule-Walker fixed point processes do not combine to construct Levinson-Durbin sequences.