A Generalized Threshold Model for Analyzing Non-Normal Nonlinear Time Series

Noelle I. Samia

Department of Statistics
Northwestern University

Abstract:

The open-loop Threshold Model proposed by Tong (1990) is a stochastic piecewise-linear regression model useful for modeling conditionally normal response time-series data. However, in many applications, the response variable is conditionally non-normal, e.g. Poisson or binomially distributed. We generalize the open-loop Threshold Model by introducing the Generalized Threshold Model (GTM). Specifically, it is assumed that the conditional probability distribution of the response variable belongs to the exponential family, and the conditional mean response is linked to some piecewise-linear stochastic regression function.

We study maximum likelihood estimation of the GTM and its large-sample properties. Under suitable regularity conditions including mixing and moment conditions as well as the discontinuity of the stochastic regression function, the maximum likelihood estimator of the threshold parameter is T-consistent (where T is the sample size) and is asymptotically independent of the estimators of the regression parameters and the delay parameter. Indeed, the super-consistency of the threshold estimator is due to the discontinuity of the conditional mean function; see Chan (1993) and Chan and Tsay (1998). The asymptotic joint distribution of the regression parameters is equal to that obtained from fitting the associated generalized linear model in the GTM with known true threshold and delay.

We illustrate the GTM with a real application on the annual number of human plague cases in Kazakhstan. The fitted GTM casts new lights on which biotic and climatic factors affect the human plague prevalence rate, and highlights the central role of the lag-1 of the flea density as the threshold variable.

This is based on a joint work with Professor Kung-Sik Chan (University of Iowa).