Espousing Classical Statistics with Modern Computation: Sufficiency, Ancillarity and An Interweaving Generation of MCMC

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Abstract:

Since the seminal work of Gelfand and Smith (1990), the past 15 years or so have witnessed a host of techniques designed to improve the speed of the Gibbs sampler and more generally MCMC in routine applications. Re-parametrizations, or variable transformations, are known to be a key to efficient implementation. However, the majority of proposals to date focus on either a single transformation or straightforward combinations of several transformations, as with Meng and van Dyk's (1997, 1999, 2001) efficient Data Augmentation (DA) strategy. Here we demonstrate that by *interweaving* two specific kinds of (one-to-one) Das or transformations, we can gain considerable speed in convergence and still retain simplicity in implementation. Simplicity is due to choosing transformations via *sufficiency* and *ancillarity*, two familiar classical concepts. In addition, by using *conditional* sufficiency and ancillarity, we can interweave different transformations for different steps (e.g., Gibbs steps) within each iteration, and thereby the proposed strategy provides a fairly general recipe for constructing a new generation of efficient algorithms for complicated applications, such as generalized linear mixed models. To demonstrate this empirically, we conduct posterior simulation under a parameter driven Poisson time series model (Cox 1981). The failure of several existing methods for fitting this model to a Chandra X-ray data set motivated us to develop this new strategy. A historical polio incidence data set (Zeger 1988) and simulated data are also used for empirical demonstration. Theoretically, we show that, in addition to being robust, under certain conditions this interweaving strategy leads to the optimal algorithm among a broad class of DA schemes. These findings also suggest several open questions, including a full explanation why interweaving sufficiency and ancillarity, two classic notions rarely associated with modern computation, can lead to such successes for MCMC.

Keywords: ancillary augmentation; convergence rate; data augmentation; EM; GLMM; MCMC; missing data; latent variables; parameter-driven model; sufficient augmentation, time series.

(joint work with Yaming Yu)