

Generalized Affine Models

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Abstract:

Let x_t be a stationary process and define $\psi_t(u)$ as the conditional cumulant function of x_t , i.e., $\psi_t(u) = \log [E [\exp(ux_{t+1}) | x_\tau, \tau \leq t]]$. This paper studies the processes characterized by $\psi_t(u) = \omega(u) + \alpha(x_t, u) + \beta(u)\psi_{t-1}(u)$, where $\omega(\cdot)$, $\alpha(\cdot)$, and $\beta(\cdot)$ are real functions. When $\alpha(x_t, u)$ is an affine function of x_t , we call x_t a generalized affine process; otherwise, we call it a generalized non-affine process. Generalized affine models include popular financial time series models including ARMA, Heston and Nandi (2000) GARCH model, affine, and log-ACD. Generalized non-affine models include non-linear models and the traditional GARCH models. We study the statistical properties of the generalized affine models and propose estimation methods. For generalized affine examples of stock returns models, we derive analytical formulas of option prices, the term structure of Value-at-Risk and expected shortfall. Likewise, we derive analytical forms of the term structure of interest rates when the short term of interest rate is a generalized affine model. We provide an empirical example of the term structure of VaR.