Some Asymptotic Aspects in the Prediction of Long Memory Time Series

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Abstract:

This talk assesses the performance of finite-sample linear predictors as compared to forecasts based on the infinite past in the context of long-range dependent time series. We introduce a semi-parametric class of long-memory processes which includes the popular fractional autoregressive moving average model, establishing some convergence results for the autoregressive expansion based on a finite number of past observations. In particular, we prove that the rest of the autoregressive expansion satisfies a central limit theorem whose rate of convergence does not depend on the long-memory parameter of the process but whose variance depends on this parameter. As a consequence, the mean square error of the forecasts obtained by truncating the autoregressive expansion depends only on the long-memory parameter and it is independent of any short-memory component of the model.

Furthermore, we study the effect of the mean estimation of the series on the convergence of the autoregressive expansion. We show that estimating the mean does not affect the rate of convergence but modifies its variance. Surprisingly, the variance can even be smaller in some cases.

Simulation results with fractional autoregressive moving average time series show that the asymptotic rates obtained from the theory are very close to the rates reached in practice, even with a relative small number of observations. In this work, we study one-step forecasts as well as multi-step forecasts.