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Study Guide

for the Ph.D. Comprehensive Exam

Probability (22S:203)

The following texts are suggested as references:

Shorack, *Probability for Statisticians*

Karr, *Probability*

Durrett, *Probability: Theory and Examples*

Ash, *Real Analysis and Probability*

Ross, *Introduction to Probability Models*

Topics

1. Measures, probability measures, sigma-fields, generated sigma-fields, probability spaces, Borel sets, construction of measures, monotone properties of measures, liminf and limsup of sets, Dynkin's $\pi - \lambda$ theorem, monotone class theorem, completion of probability spaces, Lebesque sets, Cartheodory extension theorem, Lebesgue-Stieltjes measures

2. Measurable functions, random variables, random vectors, induced probability measures, cumulative distribution functions

3. Integration, Lebesgue integral, Riemann integral, Lebesgue-Stieltjes integral, Riemann-Stieltjes integral, expectations

4. Standard convergence theorems: Fatou's lemma, monotone convergence theorem, dominated convergence theorem

5. Derivatives via signed measures; decomposition of signed measures, the Radon-Nikodym theorem, the fundamental theorem of calculus, probability density functions, change of variable integration theorems, absolute continuity of measures

6. Finite dimensional product spaces, random vectors, measurability of random vectors, Fubini's theorem, evaluation of integrals over product spaces, Tonelli's theorem

7. Modes of convergence; almost sure convergence, convergence in probability, convergence in distribution, L^P convergence; relationships among modes of convergence; moments, uniform integrability; inequalities (Jensen, Holder, Schwarz, Markov, and Chebyschev)