

Study Guide
for the M.S. Comprehensive Exam

Mathematical Statistics (22S:193-194-195)

The following texts are suggested as references:

HC: Hogg and Craig, *Introduction to Mathematical Statistics*, 5th ed (Prentice Hall)

R1: Ross, *A First Course in Probability*, 5th ed (Macmillan)

R2: Ross, *An Introduction to Probability Models*, 5th ed (Academic Press)

R3: Ross, *Stochastic Processes*, 2nd ed (Wiley)

CB: Casella and Berger, *Statistical Inference* (Wadsworth)

Topics (number in parentheses indicates chapter):

1. Probability Spaces: HC(1), R1(2), CB(1), R2(1)
 - a. Sample Spaces and Events
 - b. Axioms
 - c. Properties
2. Combinatorial Analysis: HC(1), R1(2), CB(1)
 - a. Sampling with and without Replacement
 - b. Permutations and Combinations
3. Conditional Probability and Independence: HC(1), R1(3), R2(1,3), CB(1)
 - a. Definition and Properties
 - b. Bayes Theorem
 - c. Independence
4. Random Variables: HC(1), R1(4,5), R2(2), CB(1)
 - a. Definition
 - b. Discrete, Continuous, and Mixed
 - c. Cumulative Distribution Function
 - d. Mass Functions and Densities

5. Properties of Distributions: HC(1), R1(7), CB(2)
 - a. Measures of Location (mean, median, quantiles)
 - b. Measures of Dispersion (variance, interquartile range, ...)
 - c. Moment-generating function (mgf)

6. Special Discrete Distributions: HC(3), R1(4), R2(2), CB(3)
 - a. Binomial
 - b. Hypergeometric
 - c. Poisson
 - d. Geometric and Negative Binomial
 - e. Moments (mgf) of these

7. Special Continuous Distributions: HC(3), R1(5), R2(2), CB(3)
 - a. Normal
 - b. Gamma (including Exponential and Chi-square)
 - c. Uniform
 - d. Beta
 - e. Moments (mgf) of these

8. Multivariate Distributions: HC(2,3), R1(6), R2(2), CB(4)
 - a. Definition
 - b. Univariate Normal
 - c. Bivariate Normal
 - d. Multinomial
 - e. Joint and Marginal Distributions
 - f. Conditional Distributions and Independence
 - g. Covariance and Correlation
 - h. Regression Functions

9. Generating Functions: HC(1,2,3,4), R1(7), R2(2), CB(2,5)
 - a. Moment Generating Functions
 - b. Obtaining Moments from Generating Functions
 - c. Moment Generating Functions of Sums of Independent Random Variables

10. Functions of Random Variables: HC(4), R1(7), CB(5)
 - a. Distributions Function Technique
 - b. Moment Generating Function Technique
 - c. Expectations of Functions of Random Variables
 - d. Linear Combinations of Random Variables
 - e. Change of Variable Technique
 - f. Sampling from the Normal Distribution (includes the t- and F-distributions)
 - g. Order Statistics (definition and distributions)

11. Convergence Concepts: HC(5), R1(8), R2(2), CB(5)

- a. Convergence in Probability
- b. Convergence in Distribution
- c. Laws of Large Numbers
- d. Central Limit Theorem

12. Estimation: HC(6,7,8), CB(7,9,10)

- a. Point Estimation (Maximum Likelihood, Method of Moments, Unbiased Estimators – including Minimum Variances, Bayes Estimation)
- b. Interval Estimation (Confidence Intervals for Proportions, Means, Variance, and Pivotal Quantities)
- c. Bayesian Methods
- d. Methods of Evaluating Estimators (Mean Square Error, Consistency)

13. Statistical Hypotheses: HC(6,9,10), CB(8)

- a. Definition of Tests of Statistical Hypotheses
- b. Neyman-Pearson Lemma
- c. Power Functions
- d. Uniformly Most Powerful Tests
- e. Likelihood Ratio Tests
- f. Sequential probability ratio test

14. Sufficient Statistics: HC(7), CB(6,7)

- a. Definition
- b. Rao-Blackwell Theorem
- c. Completeness and Uniqueness
- d. Exponential Families
- e. Sufficiency and Independence

15. The Poisson Process: R1(9), R2(5), R3(2), HC(3)

- a. The Poisson Process
- b. Interarrival and Waiting Time Distributions
- c. Non-homogeneous Poisson Process
- d. Compound Poisson Processes

16. Markov Chains: R1(9), R2(4), R3(4)

- a. Definition and examples
- b. Chapman-Kolmogorov Equations
- c. Classification of States
- d. Limit Theorems
- e. Transitions Among Classes, Mean Time in Transient States
- f. Branching Processes
- g. Time-Reversible Markov Chains