

**22S:30/105**  
**Statistical Methods and**  
**Computing**

**Measures of Center, continued**  
**Measures of Dispersion**

Lecture 3  
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Example:

- An ecological study of a habitat in which 10 rare species of bird are known to have lived as of 1990
- In 1999, a naturalist is sent to spend a day in the area and to record any members of these 10 species that she observes
- A variable is coded as follows:
  - 1 = at least one member of the species was observed
  - 0 = no members of the species were observed

The mean is meaningful only for quantitative data (either discrete or continuous).

- Example regarding a discrete variable: We hear reports such as that the average number of children per family is 1.9.
- The mean is not meaningful for nominal or ordinal data.

Exception: if a binary variable is coded as 0 and 1.

Then the arithmetic mean is the proportion of observations in the dataset that have value 1.

species	observed?
1	1
2	0
3	1
4	1
5	1
6	0
7	1
8	1
9	1
10	1

- The mean

$$\bar{x} = \frac{8}{10} = .8$$

indicates that 80% of the species were observed.

## The median

The median is the 50th percentile of a set of observations.

- Values must be sorted from smallest to largest.
- If the number of observations is odd, then the median is the middle value.

75 80 82 88 95

The median is 82.

- If the number of observations is even, then the usual way to define the median is as the **mean** of the **two** middle values.

75 80 82 88 95 97

The median is  $\frac{82+88}{2} = 85$ .

The median can be used as a measure of center for **ordinal** data as well as for discrete and continuous data.

Example: The NYC poll

city1yr	Frequency	Percent	Cumulative Frequency
Worse	593	61.64	593
Same	252	26.20	845
Better	111	11.54	956

- 956 people answered this question regarding whether they thought the condition of the city in June, 2003, was better, worse, or the same as one year earlier.
- If the values are sorted from smallest to largest (Worse, Same, Better), then the median will be the average of the 478th and 479th values.
- We can use the cumulative frequencies in the table to figure out what these have to be. They are both in category "Worse."
- Thus the median is Worse.

The median is **not** strongly affected by a few extreme values in the dataset.

Example 1:

75 80 82 88 95

- mean = 84
- median = 82

Example 2:

25 80 82 88 95

- mean = 74
- median = 82

The median is *robust* to extreme values.

## The mode

- The mode of a set of values is the value that occurs most frequently.
- Example: in the NYC poll data, the mode of the "city1yr" variable is Worse.
- Example: There is no mode in the birthweights data, because no value occurs more than once.
- There may be more than one mode in a set of values.
- The mode may be reported for **all** types of data.

When is each measure of central tendency appropriate?

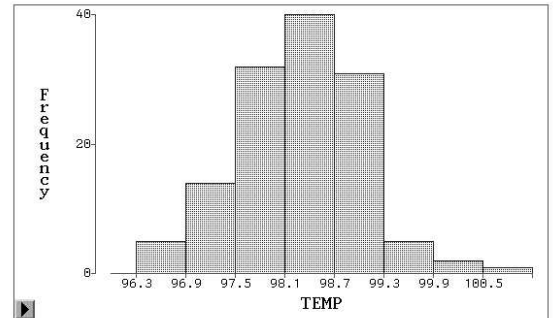
**Depending on data type**

- Nominal data
  - mode only
  - possible exception: binary data coded 0 and 1
- Ordinal data
  - mode or median
- Quantitative data
  - mean, median, or mode

**Depending on the shape of the distribution**  
of values (quantitative variables)

- if the shape is approximately symmetric and has only one mode
  - mean and median will be close in value
  - mean is typically reported

Example: the body temperature data

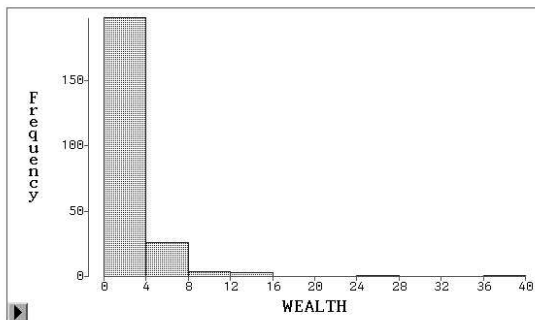


From a statistical computer package:

- mean = 98.24
- median = 98.3

- if the distribution is highly skewed
  - if skewed to the right, mean will be larger than median
  - if skewed to the left, mean will be smaller than median
  - mean may not be a “typical” value

Example: the billionaire data

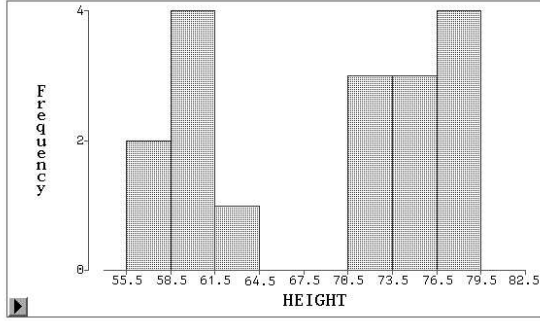


From a statistical computer package:

- mean = 2.7 billion
- median = 1.8 billion

- if the distribution has more than one mode
  - neither the mean nor the median may be representative values
  - may be best to report all modes and/or to display a graph
  - may occur if two or more different subgroups are represented in the sample

Example:

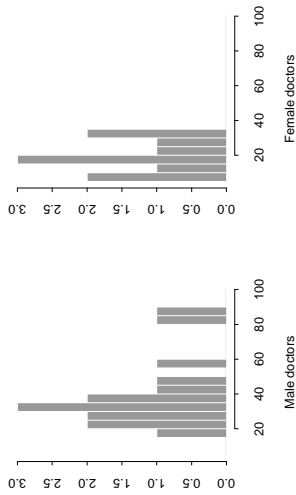


From a statistical computer package:

- mean = 69.0
- median = 72.0

In getting the “overall picture” of quantitative data, the spread is just as important as the center of the data.

Number of Caesarian Sections Performed in a Single Year by Swiss Doctors



### Numerical measures of dispersion

- the range
- the interquartile range
- the standard deviation

## The range

- The range is the difference between the largest and the smallest observations.
- For the male Swiss doctors,
  - largest value = 86
  - smallest value = 20
  - range =  $86 - 20 = 66$
- For the female Swiss doctors,
  - largest value = 33
  - smallest value = 5
  - range =  $33 - 5 = 28$

## The quartiles and the interquartile range

- The *first quartile* is the same as the 25th percentile
  - one quarter of the observations in a dataset have values less than or equal to the 1st quartile, and the other three quarters have values greater than or equal to the first quartile
- The *third quartile* is the same as the 75th percentile
  - three quarters of the observations in a dataset have values less than or equal to the 3rd quartile, and the other one quarter have values greater than or equal to the 3rd quartile

The range shows the full spread of the data, but may be exaggerated if the largest and/or smallest values are atypical (outliers)

- Example: the 1992 billionaire data
  - With Bill Gates:  
range =  $37 - 1 = 36$  billion
  - If Bill were deleted:  
range =  $24 - 1 = 23$  billion
- Example: the male Swiss doctors data
  - With the largest two values  
range =  $86 - 20 = 66$  billion
  - If the two largest values were deleted:  
range =  $59 - 20 = 39$  billion
- So additional measures are needed to give a more complete picture of the spread of values.

- The interquartile range (IQR) is the difference between the 3rd and 1st quartiles
- For the male Swiss doctors,
  - third quartile = 50
  - first quartile = 27
  - IQR =  $50 - 27 = 23$
- For the female Swiss doctors,
  - third quartile = 29
  - first quartile = 14
  - IQR =  $29 - 14 = 15$
- For the 1992 billionaires,
  - third quartile = 3 billion
  - first quartile = 1.3 billion
  - IQR =  $3 - 1.3 = 1.7$  billion

The IQR is considered less sensitive to outliers than the range.

- Example: the 1992 billionaire data
  - With Bill Gates:  
IQR = 3 – 1.3 = 1.7 billion
  - If Bill were deleted:  
IQR = 2.9 – 1.3 = 1.6 billion
- However, in a small dataset, deletion of a few outliers may affect the IQR substantially.
- Example: the male Swiss doctors
  - IQR with the two largest values included:  
IQR = 50 – 27 = 23
  - IQR with the two largest values deleted:  
IQR = 37 – 27 = 10

The five-number summary for the billionaire data may be extracted from the following computer output:

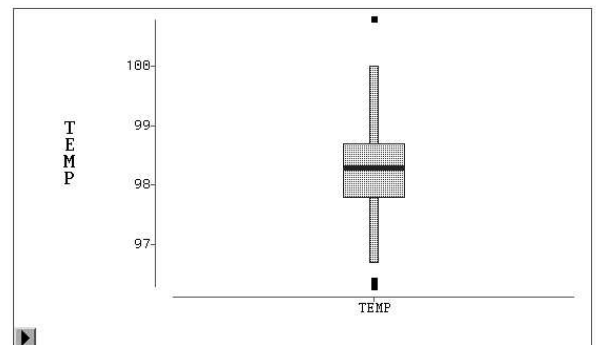
Quantiles(Def=5)			
100% Max	37	99%	14
75% Q3	3	95%	6.2
50% Med	1.8	90%	4.5
25% Q1	1.3	10%	1.1
0% Min	1	5%	1
		1%	1
Range	36		
Q3-Q1	1.7		
Mode	1		

## The five-number summary

- The five-number summary provides a reasonably-complete numeric summary of the center and dispersion of a set of values.
- The five-number summary consists of
  - the minimum value
  - the first quartile
  - the median
  - the third quartile
  - the maximum value

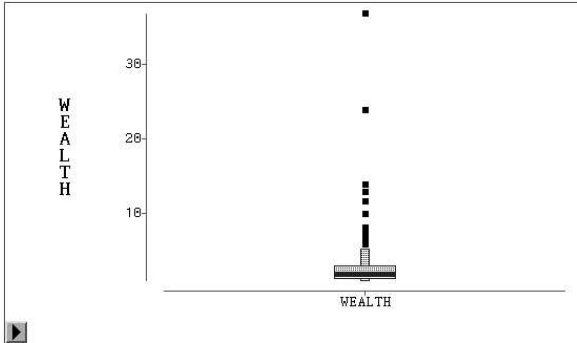
## Boxplots

- are used to summarize the distribution of a continuous variable



- box extends from 1st quartile to 3rd quartile of data
- line in middle of box marks 50th percentile

- “whiskers” sticking out of box extend to *adjacent values*
  - adjacent values are most extreme observations that are not farther away from the edge of the box than 1.5 times the height of the box
- points farther out than the adjacent values are considered *outliers*
  - represented by circles or squares
  - probably are not typical of the rest of the data



An idea that won't work for measuring the spread: take the average of the “deviations” of the individual observations from the mean.

Observed Value	Deviation from mean	Squared deviation
75	$75 - 84 = -9$	$(-9)^2 = 81$
80	$80 - 84 = -4$	$(-4)^2 = 16$
82	$82 - 84 = -2$	$(-2)^2 = 4$
88	$88 - 84 = 4$	$4^2 = 16$
95	$95 - 84 = 11$	$11^2 = 121$
sum	0	238

## The standard deviation

- The standard deviation measures spread by looking at how far the observations are from their mean.
- Example: quiz scores

75 80 82 88 95

The mean is

$$\bar{x} = \frac{75 + 80 + 82 + 88 + 95}{5} = 84$$

points.

- We want a measure of typical distance between an individual value and this mean.

Because the sum of the deviations is always 0, the average deviation is always 0!

Solution: Square the individual deviations before adding them up!

The variance and the standard deviation

- The variance  $s^2$  is the sum of the squared deviations divided by one less than the number of observations.

$$\begin{aligned} s^2 &= \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n - 1} \\ &= \frac{238}{4} = 59.5 \end{aligned}$$

- We can think of the variance roughly as the average of the squared deviations.
- The standard deviation is the square root of the variance.  
 $s = \sqrt{59.5} = 7.71$  points.

## Facts about the standard deviation

$s$

- $s$  measures the spread of values around the *mean*
  - thus  $s$  should be used as a measure of dispersion only when mean has been chosen as the measure of center
- $s$  is always greater than 0 unless all the observations have the same value
- $s$  has same units of measurement as original observations
- $s$  is sensitive to extreme observations
  - like the mean
- $s$  is the most commonly-used measure of dispersion (is often used when it is not the best choice!)

The mean and standard deviation together provide a reasonable numeric summary of a set of values if the distribution is approximately **symmetric**.

- Example: the body temperature data

Variable	N	Mean	Std Dev
TEMP	130	98.2492308	0.7331832

- Example of inappropriate use of  $\bar{x}$  and  $s$  to summarize a distribution: the billionaire data

Analysis Variable : WLTH

N	Mean	Std Dev
233	2.6815451	3.3188403