PRACTICE PROBLEMS for FINAL 2006 22S:30/105, Statistical Methods and Computing Spring 2005, Instructor: Cowles

## Final Exam

Name: $\qquad$ Course no. (30 or 105) $\qquad$

## Secret number for posting grade:

$\qquad$
Note: The number of points for each question is indicated in parentheses.

## 1 General questions

1. Mr. Rex Boggs from Australia weighed the bar of soap in his shower stall each morning before showering. The weight went down as the soap was used. On some days he forgot to weigh the soap. The scatterplot below shows the weight (in grams) versus the day of measurement (numbered 1 through 21).

(a) (1) Based on the scatterplot, the sample correlation is (circle one)
i. close to 1
ii. positive but not close to 1
iii. close to 0
iv. negative but not close to -1
v. close to -1
(b) (1) Briefly explain your answer
2. For a biology project, you measure the thorax length (in millimeters) and the weight (in milligrams) of 12 bees of the same species. What units of measurement do each of the following have:
(a) (1) the sample mean weight
(b) (1) the first quartile of weight
(c) (1) the standard deviation of weight
(d) (1) the correlation between thorax length and weight
3. Digoxin is a drug often prescribed for patients with heart disease. It is taken in pill form, and patients are instructed to drink a full glass of water when they take their digoxin.
Researchers (Parker et al., Pharmacotherapy, 2003) were interested in whether the concentration of digoxin in the bloodstream would be higher if people drank grapefruit juice instead of water when they took their digoxin.
Seven volunteers participated in the study. Subjects took digoxin with water for 2 weeks, no digoxin for 2 weeks, and digoxin with grapefruit juice for 2 weeks. The response variable - peak concentration of digoxin in the blood plasma (Cmax) was measured on each patient during the water period and again during the grapefruit juice period. Cmax is a continuous quantitative variable.
We wish to determine whether their data give evidence at the .05 significance level that Cmax is higher when digoxin is taken with grapefruit juice than when it is taken with water.
(a) (1) Which type of problem is this? (Circle one)
i. single sample
ii. paired sample
iii. two independent sample
iv. none of the above
(b) (2) Of the statistical tests that we have studied, the one most likely to be useful for addressing this problem is a paired $t$-test. Which of the following assumptions need to be met for the paired $t$-test to give reliable results in this problem? (Circle as many as apply.)
i. The distribution of Cmax must be approximately normal in the population of all people who take digoxin with water and in the population of all people who take digoxin with grapefruit juice.
ii. The population distribution of differences between Cmax when digoxin is taken with water and Cmax when digoxin is taken with grapefruit juice must be approximately normal.
iii. The population standard deviations of Cmax must be approximately equal in the population of all people who take digoxin with water and in the population of all people who take digoxin with grapefruit juice.
iv. $n p$ and $n(1-p)$ must both be greater than or equal to 5 .
v . none of the above.
4. Psychiatrists wish to determine the effects of different types of lighting (full spectrum light, regular fluorescent light, and regular incandescent light) and supplementation with Omega 3 fatty acids on depression. Forty eight people who have been diagnosed with mild depression are recruited into the study. They are randomly assigned to six groups, with 8 people in each group. Each subject is given a light bulb to install in the place where he/she spends the most time during each day. Each subject is also given a bottle of pills, of which they are to take one each day. The groups receive the following:

- full spectrum light; Omega 3 fatty acid supplements
- regular fluorescent light; Omega 3 fatty acid supplements
- regular incandescent light; Omega 3 fatty acid supplements
- full spectrum light; placebo
- regular fluorescent light; placebo
- regular incandescent light; placebo

The subjects are not told which type of lightbulb they have been given and whether their bottle of pills is real supplements or placebos.
The subjects are given a a written depression inventory test at the beginning of the study and again after a month on the light/supplements regimen. The researchers are interested in whether the changes in depression scores are different in the different groups.
(a) (1) Is this an experiment or an observational study?
(b) (1)What are the experimental units?
(c) (1) What are the factors?
(d) (1) What are the levels?
(e) (1) What are the treatments?
(f) (1) What is the response variable?
5. A historian examining British colonial records for the Gold Coast in Africa suspects hat the death rate was higher among African miners than among European miners. In the year 1936 there were

## 223 deaths among 33,809 African miners

7 deaths among 1541 European miners
in the Gold Coast. (Data courtesy of Raymond Dumett, Purdue University).
Consider this year as a sample from the prewar era in Africa. We wish to determine whether the data provides good evidence that the proportion of African miners who died during a year was higher than the proportion of European miners who died.
(a) (2) State the null and alternative hypotheses, using conventional symbols.
(b) (3) Calculate a test statistic. Show your work and give a numeric result.
(c) (1) Give a p-value as exact as the tables in the text allow. (numeric result)
(d) (2) State your conclusion in terms of this application. (If you could not get the p-value in the preceding question, pretend that it was .008 and answer this question accordingly.)
(e) (3) Give a $95 \%$ confidence interval for the difference between the proporation of African miners who would die in a year and the proportion of European miners who would die in a year. (numeric answer)
6. A car salesman would like to estimate the proportion of all UI faculty who have not purchased a car in the last 5 years. He will select a simple random sample of UI faculty and will ask each person in the sample whether he or she has purchased a car in the last 5 years. The salesman wants to calculate a $90 \%$ confidence interval with margin of error no greater than 0.03
(a) (2) How large a simple random sample of UI faculty will he need if he is pretty sure that the true population proportion is close to .15 ?
b) (2) How large a simple random sample of UI faculty will he need if he has no preliminary idea about the population proportion?
(c) (2) The total number of faculty at the UI is about 1200. If the car salesman does obtain a sample of the size you calculated in the second part of this problem, should he use normal approximations to calculate his confidence interval? (yes/no) Why or why not?
7. For each of the following variables, state which data type it is (binary, nominal, ordinal, quantitative continuous, quantitative discreet).
(a) (0.5) hair color (evaluated on a sample of human beings)
(b) (0.5) boiling temperature of water (evaluated at a number of different elevations in the mountaints)
8. Every spring, Nenana, Alaska, hosts a contest in which participants try to guess the exact minute that a wooden stand placed on the frozen Tanana River will fall through the breaking ice. The contest started in 1917 as entertainment for railroad engineers. It has grown into an event in which hundreds of thousands of entrants enter their guesses on the Internet and compete for prizes of more than $\$ 300,000$. Because so much money depends on the time of ice breakup, it has been recorded to the nearest minutie with great accuracy ever since 1917.An article in Science ("Climate Change in Nontraditional Datasets," Oct. 2001, p. 811) used the data to investigate global
warming by asking the question whether ice breakup had tended to occur earlier over time.
The dataset available to us contains two variables

- year
- julian - the number of days from midnight on Jan 1 until the time of ice breakup Refer to the SAS output provided to answer the following questions.
(a) (1) The null hypothesis is that there is no linear relationship between year and time of ice breakup. Write this null hypothesis as a statement about a population parameter. Use conventional symbols.
(b) (1) The alternative hypothesis is that time of ice breakup decreases linearly over time. Write this alternative hypothesis as a statement about a population parameter. Use conventional symbols.
(c) (1) Give a point estimate and a $95 \%$ confidence interval for the population slope (numeric answers).
(d) (2) Does your answer to the preceding question provide evidence in favor of the alternative hypothesis? (yes/no) Explain briefly. (If you could not answer the previous question, pretend that the point estimate is -0.11 and the confidence interval is ( $-0.21,-0.01$ ) and answer this question accordingly.
wer)
(e) (0.5) What is the estimated value of the standard deviation of points around the regression line? (numeric answer)

9. (1.5) What is the p-value for the one-sided test of no linear relationship between year and time of breakup?
10. (1.5) Use the estimated regression equation to predict the time of breakup for this year (2005). Show your calculation
11. (1) On the SAS output, circle the numbers that provide the endpoints of the interval in which you are $95 \%$ confident that breakup in 2005 time would lie. Be sure to put your name on the SAS output.
12. (1) What proportion of the variability in time of breakup is explained by year? (numeric answer)
13. (0.5) What is the estimated value of the standard deviation of points around the regression line? (numeric answer)

| Obs | year | julian |
| :---: | :---: | :---: |
|  |  |  |
| 1 | 1917 | 120.480 |
| 2 | 1918 | 131.398 |
| 3 | 1919 | 123.607 |
| many | lines | omitted |
| 77 | 1993 | 113.543 |
| 78 | 1994 | 119.959 |
| 79 | 1995 | 116.557 |
| 80 | 1996 | 126.523 |
| 81 | 1997 | 120.437 |
| 82 | 1998 | 110.705 |
| 83 | 1999 | 119.908 |
| 84 | 2000 | 122.450 |
| 85 | 2001 | 128.542 |
| 86 | 2002 | 127.894 |
| 87 | 2003 | 119.766 |
| 88 | 2005 | . |

The REG Procedure Model: MODEL1
Dependent Variable: julian
$\begin{array}{lr}\text { Number of Observations Read } & 88 \\ \text { Number of Observations Used } & 87 \\ \text { Number of Observations with Missing Values } & 1\end{array}$

## Analysis of Variance

| Source | DF | Sum of <br> Squares | Mean <br> Square | F Value | Pr > F |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Model | 1 | 238.06059 | 238.06059 | 7.26 | 0.0085 |
| Error | 85 | 2787.93204 | 32.79920 |  |  |
| Corrected Total | 86 | 3025.99262 |  |  |  |
|  |  |  |  |  |  |
|  |  |  |  |  |  |
| $\quad$ Root MSE |  | 5.72706 | R-Square | 0.0787 |  |
| $\quad$ Dependent Mean | 125.54431 | Adj R-Sq | 0.0678 |  |  |
| $\quad$ Coeff Var | 4.56178 |  |  |  |  |

Parameter Estimates

| Variable | DF | Parameter <br> Estimate | Standard Error | t Value | $\operatorname{Pr}>\|t\|$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Intercept | 1 | 254.64848 | 47.92518 | 5.31 | <. 0001 |
| year | 1 | -0.06587 | 0.02445 | -2.69 | 0.0085 |
| Parameter Estimates |  |  |  |  |  |
|  | Variable | DF 95\% Confidence Limits |  |  |  |
|  | Intercept | 1 | 159.36037 | 349.93658 |  |
|  | year | 1 | -0.11448 | -0.01726 |  |

Output Statistics

| Obs year |  | Dependent Predicted Std Error |  |  | 95\% CL Mean |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Variable | Val | Mean Predict |  |  |
| 1 | 1917 | 120.4795 | 128.3767 | 1.2175 | 125.9560 | 130.7974 |
| 2 | 1918 | 131.3983 | 128.3108 | 1.1965 | 125.9320 | 130.6897 |
| 3 | 1919 | 123.6066 | 128.2450 | 1.1755 | 125.9077 | 130.5822 |
| 4 | 1920 | 132.4490 | 128.1791 | 1.1548 | 125.8831 | 130.4751 |
| 5 | 1921 | 131.2795 | 128.1132 | 1.1341 | 125.8583 | 130.3682 |
| 6 | 1922 | 132.5559 | 128.0474 | 1.1136 | 125.8331 | 130.2616 |
| 7 | 1923 | 129.0837 | 127.9815 | 1.0933 | 125.8077 | 130.1553 |
| 8 | 1924 | 132.6323 | 127.9156 | 1.0732 | 125.7818 | 130.0494 |
| 9 | 1925 | 127.7726 | 127.8497 | 1.0532 | 125.7556 | 129.9438 |
| 10 | 1926 | 116.6691 | 127.7839 | 1.0335 | 125.7291 | 129.8387 |
| 11 | 1927 | 133.2378 | 127.7180 | 1.0139 | 125.7021 | 129.7339 |
| 12 | 1928 | 127.6844 | 127.6521 | 0.9946 | 125.6747 | 129.6296 |
| 13 | 1929 | 125.6538 | 127.5863 | 0.9754 | 125.6468 | 129.5257 |
| 14 | 1930 | 128.7941 | 127.5204 | 0.9566 | 125.6185 | 129.4223 |
| 15 | 1931 | 130.3913 | 127.4545 | 0.9379 | 125.5896 | 129.3194 |
| 16 | 1932 | 122.4274 | 127.3887 | 0.9196 | 125.5602 | 129.2171 |
| 17 | 1933 | 128.8128 | 127.3228 | 0.9015 | 125.5303 | 129.1153 |
| 18 | 1934 | 120.5885 | 127.2569 | 0.8838 | 125.4997 | 129.0142 |
| 19 | 1935 | 135.5642 | 127.1910 | 0.8664 | 125.4685 | 128.9136 |
| 20 | 1936 | 121.5406 | 127.1252 | 0.8493 | 125.4365 | 128.8138 |
| 21 | 1937 | 132.8365 | 127.0593 | 0.8326 | 125.4039 | 128.7148 |
| 22 | 1938 | 126.8434 | 126.9934 | 0.8163 | 125.3704 | 128.6164 |
| 23 | 1939 | 119.5601 | 126.9276 | 0.8004 | 125.3362 | 128.5190 |
| 24 | 1940 | 111.6441 | 126.8617 | 0.7849 | 125.3011 | 128.4224 |
| 25 | 1941 | 123.0767 | 126.7958 | 0.7699 | 125.2650 | 128.3267 |
| 26 | 1942 | 120.5615 | 126.7300 | 0.7554 | 125.2280 | 128.2320 |
| 27 | 1943 | 118.8073 | 126.6641 | 0.7415 | 125.1899 | 128.1383 |
| 28 | 1944 | 125.5892 | 126.5982 | 0.7280 | 125.1507 | 128.0458 |
| 29 | 1945 | 136.4038 | 126.5324 | 0.7152 | 125.1104 | 127.9544 |
| 30 | 1946 | 125.6948 | 126.4665 | 0.7030 | 125.0688 | 127.8642 |
| 31 | 1947 | 123.7455 | 126.4006 | 0.6914 | 125.0259 | 127.7753 |
| 32 | 1948 | 134.4677 | 126.3347 | 0.6805 | 124.9817 | 127.6878 |
| 33 | 1949 | 134.5274 | 126.2689 | 0.6703 | 124.9361 | 127.6017 |
| 34 | 1950 | 126.6767 | 126.2030 | 0.6609 | 124.8890 | 127.5170 |
| 35 | 1951 | 120.7462 | 126.1371 | 0.6522 | 124.8403 | 127.4340 |
| 36 | 1952 | 133.7115 | 126.0713 | 0.6444 | 124.7900 | 127.3525 |
| 37 | 1953 | 119.6628 | 126.0054 | 0.6374 | 124.7381 | 127.2727 |
| 38 | 1954 | 126.7510 | 125.9395 | 0.6313 | 124.6844 | 127.1947 |
| 39 | 1955 | 129.5927 | 125.8737 | 0.6261 | 124.6289 | 127.1184 |
| 40 | 1956 | 122.9753 | 125.8078 | 0.6217 | 124.5716 | 127.044 |

> 958 119.6226 125.676 $\begin{array}{llll}1959 & 128.4767 & 125.6102\end{array}$ $\begin{array}{lll}1960 & 123.8003 & 125.5443\end{array}$ $1961 \quad 125.4802 \quad 125.4784$ $\begin{array}{lll}1962 & 132.9747 & 125.4126\end{array}$ $\begin{array}{lll}1963 & 125.7677 & 125.3467\end{array}$ $1964 \quad 141.4872 \quad 125.2808$
$\begin{array}{lll}0.6184 & 124.5124 & 126.9714\end{array}$ $\begin{array}{lll}0.6159 & 124.4514 & 126.9007\end{array}$ $0.6145 \quad 124.3884126 .8320$ $\begin{array}{lll}0.6140 & 124.3235 & 126.7651\end{array}$ $\begin{array}{llll}0.6145 & 124.2567 & 126.7002\end{array}$ $\begin{array}{lll}0.6145 & 124.2567 & 126.7002\end{array}$ $\begin{array}{lll}0.6159 & 124.1879 & 126.6372\end{array}$ $\begin{array}{lll}0.6184 & 124.1172 & 126.5762 \\ 0.6217 & 124.0446 & 126.5170\end{array}$

| Output Statistics |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 95\% CL Predict |  | Residual |
| 1 | 1917 | 116.7353 | 140.0181 | -7.8972 |
| 2 | 1918 | 116.6781 | 139.9436 | 3.0875 |
| 3 | 1919 | 116.6206 | 139.8693 | -4.6384 |
| 4 | 1920 | 116.5630 | 139.7952 | 4.2699 |
| 5 | 1921 | 116.5052 | 139.7213 | 3.1663 |
| 6 | 1922 | 116.4471 | 139.6476 | 4.5085 |
| 7 | 1923 | 116.3889 | 139.5741 | 1.1022 |
| 8 | 1924 | 116.3305 | 139.5007 | 4.7167 |
| 9 | 1925 | 116.2719 | 139.4276 | -0.0771 |
| 10 | 1926 | 116.2130 | 139.3547 | -11.1148 |
| 11 | 1927 | 116.1540 | 139.2820 | 5.5198 |
| 12 | 1928 | 116.0948 | 139.2095 | 0.0323 |
| 13 | 1929 | 116.0354 | 139.1372 | -1.9325 |
| 14 | 1930 | 115.9757 | 139.0651 | 1.2737 |
| 15 | 1931 | 115.9159 | 138.9932 | 2.9368 |
| 16 | 1932 | 115.8559 | 138.9214 | -4.9613 |
| 17 | 1933 | 115.7956 | 138.8499 | 1.4900 |
| 18 | 1934 | 115.7352 | 138.7786 | -6.6684 |
| 19 | 1935 | 115.6746 | 138.7075 | 8.3732 |
| 20 | 1936 | 115.6137 | 138.6366 | -5.5846 |
| 21 | 1937 | 115.5527 | 138.5659 | 5.7772 |
| 22 | 1938 | 115.4914 | 138.4955 | -0.1500 |
| 23 | 1939 | 115.4300 | 138.4252 | -7.3675 |
| 24 | 1940 | 115.3683 | 138.3551 | -15.2176 |
| 25 | 1941 | 115.3065 | 138.2852 | -3.7191 |
| 26 | 1942 | 115.2444 | 138.2155 | -6.1685 |
| 27 | 1943 | 115.1821 | 138.1461 | -7.8568 |
| 28 | 1944 | 115.1197 | 138.0768 | -1.0090 |
| 29 | 1945 | 115.0570 | 138.0077 | 9.8714 |
| 30 | 1946 | 114.9941 | 137.9389 | -0.7717 |
| 31 | 1947 | 114.9310 | 137.8702 | -2.6551 |
| 32 | 1948 | 114.8677 | 137.8018 | 8.1330 |
| 33 | 1949 | 114.8042 | 137.7335 | 8.2585 |
| 34 | 1950 | 114.7405 | 137.6655 | 0.4737 |
| 35 | 1951 | 114.6766 | 137.5977 | -5.3909 |
| 36 | 1952 | 114.6125 | 137.5301 | 7.6402 |
| 37 | 1953 | 114.5482 | 137.4626 | -6.3426 |
| 38 | 1954 | 114.4836 | 137.3954 | 0.8115 |
| 39 | 1955 | 114.4189 | 137.3284 | 3.7190 |
| 40 | 1956 | 114.3540 | 137.2616 | -2.8325 |
| 41 | 1957 | 114.2888 | 137.1950 | -0.3457 |


| ar |  | Dependent <br> Variable | Predicted <br> Value | Std Error Mean Predict | 95\% CL | Mean |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 49 | 1965 | 127.7927 | 125.2150 | 0.6261 | 123.9702 | 126.4597 |
| 50 | 1966 | 128.5080 | 125.1491 | 0.6313 | 123.8939 | 126.4043 |
| 51 | 1967 | 124.4969 | 125.0832 | 0.6374 | 123.8159 | 126.3506 |
| 52 | 1968 | 129.3934 | 125.0174 | 0.6444 | 123.7361 | 126.2986 |
| 53 | 1969 | 118.5198 | 124.9515 | 0.6522 | 123.6547 | 126.2483 |
| 54 | 1970 | 124.4427 | 124.8856 | 0.6609 | 123.5716 | 126.1997 |
| 55 | 1971 | 128.8969 | 124.8197 | 0.6703 | 123.4870 | 126.1525 |
| 56 | 1972 | 131.4976 | 124.7539 | 0.6805 | 123.4009 | 126.1069 |
| 57 | 1973 | 124.4997 | 124.6880 | 0.6914 | 123.3133 | 126.0627 |
| 58 | 1974 | 126.6559 | 124.6221 | 0.7030 | 123.2244 | 126.0198 |
| 59 | 1975 | 130.5760 | 124.5563 | 0.7152 | 123.1343 | 125.9783 |
| 60 | 1976 | 123.4524 | 124.4904 | 0.7280 | 123.0429 | 125.9379 |
| 61 | 1977 | 126.5323 | 124.4245 | 0.7415 | 122.9503 | 125.8988 |
| 62 | 1978 | 120.6378 | 124.3587 | 0.7554 | 122.8567 | 125.8607 |
| 63 | 1979 | 120.7615 | 124.2928 | 0.7699 | 122.7620 | 125.8236 |
| 64 | 1980 | 120.5531 | 124.2269 | 0.7849 | 122.6663 | 125.7876 |
| 65 | 1981 | 120.7809 | 124.1611 | 0.8004 | 122.5697 | 125.7524 |
| 66 | 1982 | 130.7337 | 124.0952 | 0.8163 | 122.4722 | 125.7182 |
| 67 | 1983 | 119.7760 | 124.0293 | 0.8326 | 122.3739 | 125.6848 |
| 68 | 1984 | 130.6483 | 123.9634 | 0.8493 | 122.2748 | 125.6521 |
| 69 | 1985 | 131.6087 | 123.8976 | 0.8664 | 122.1750 | 125.6202 |
| 70 | 1986 | 128.9517 | 123.8317 | 0.8838 | 122.0745 | 125.5889 |
| 71 | 1987 | 125.6330 | 123.7658 | 0.9015 | 121.9733 | 125.5584 |
| 72 | 1988 | 118.3858 | 123.7000 | 0.9196 | 121.8716 | 125.5284 |
| 73 | 1989 | 121.8434 | 123.6341 | 0.9379 | 121.7692 | 125.4990 |
| 74 | 1990 | 114.7219 | 123.5682 | 0.9566 | 121.6663 | 125.4701 |
| 75 | 1991 | 121.0031 | 123.5024 | 0.9754 | 121.5629 | 125.4418 |
| 76 | 1992 | 135.2684 | 123.4365 | 0.9946 | 121.4591 | 125.4139 |
| 77 | 1993 | 113.5427 | 123.3706 | 1.0139 | 121.3547 | 125.3865 |
| 78 | 1994 | 119.9594 | 123.3048 | 1.0335 | 121.2500 | 125.3595 |
| 79 | 1995 | 116.5573 | 123.2389 | 1.0532 | 121.1448 | 125.3330 |
| 80 | 1996 | 126.5226 | 123.1730 | 1.0732 | 121.0392 | 125.3068 |
| 81 | 1997 | 120.4365 | 123.1071 | 1.0933 | 120.9333 | 125.2810 |
| 82 | 1998 | 110.7045 | 123.0413 | 1.1136 | 120.8271 | 125.2555 |
| 83 | 1999 | 119.9080 | 122.9754 | 1.1341 | 120.7205 | 125.2303 |
| 84 | 2000 | 122.4497 | 122.9095 | 1.1548 | 120.6136 | 125.2055 |
| 85 | 2001 | 128.5420 | 122.8437 | 1.1755 | 120.5064 | 125.1809 |
| 86 | 2002 | 127.8941 | 122.7778 | 1.1965 | 120.3989 | 125.1567 |
| 87 | 2003 | 119.7656 | 122.7119 | 1.2175 | 120.2912 | 125.1326 |
| 88 | 2005 |  | 122.5802 | 1.2600 | 120.0750 | 125.0853 |


| Obs year | $95 \%$ CL Predict | Residual |  |  |
| :--- | ---: | ---: | ---: | ---: |
|  |  |  |  |  |
| 49 | 1965 | 113.7602 | 136.6697 | 2.5777 |
| 50 | 1966 | 113.6932 | 136.6050 | 3.3589 |
| 51 | 1967 | 113.6260 | 136.5405 | -0.5863 |
| 52 | 1968 | 113.5586 | 136.4761 | 4.3760 |
| 53 | 1969 | 113.4910 | 136.4120 | -6.4317 |
| 54 | 1970 | 113.4231 | 136.3481 | -0.4429 |
| 55 | 1971 | 113.3551 | 136.2844 | 4.0772 |
| 56 | 1972 | 113.2868 | 136.209 | 6.7437 |
| 57 | 1973 | 113.2184 | 136.1576 | -0.1883 |
| 58 | 1974 | 113.1498 | 136.0945 | 2.0338 |
| 59 | 1975 | 113.0809 | 136.0316 | 6.0197 |
| 60 | 1976 | 113.0118 | 135.9690 | -1.0380 |
| 61 | 1977 | 112.9426 | 135.9065 | 2.1078 |
| 62 | 1978 | 112.8731 | 135.8442 | -3.7209 |
| 63 | 1979 | 112.8034 | 135.7822 | -3.5313 |
| 64 | 1980 | 112.7335 | 135.7203 | -3.6738 |
| 65 | 1981 | 112.6635 | 135.6586 | -3.3802 |
| 66 | 1982 | 112.5932 | 135.5972 | 6.6385 |
| 67 | 1983 | 112.5227 | 135.5359 | -4.2533 |
| 68 | 1984 | 112.4520 | 135.4749 | 6.6849 |
| 69 | 1985 | 112.3811 | 135.4141 | 7.7111 |
| 70 | 1986 | 112.3100 | 135.3534 | 5.1200 |
| 71 | 1987 | 112.2387 | 135.2930 | 1.8672 |
| 72 | 1988 | 112.1672 | 135.2328 | -5.3142 |
| 73 | 1989 | 112.0955 | 135.1727 | -1.7907 |
| 74 | 1990 | 112.0236 | 135.1129 | -8.8463 |
| 75 | 1991 | 111.9515 | 135.0533 | -2.4993 |
| 76 | 1992 | 111.8791 | 134.9938 | 11.8319 |
| 77 | 1993 | 111.8066 | 134.9346 | -9.8279 |
| 78 | 1994 | 111.7339 | 134.8756 | -3.3454 |
| 79 | 1995 | 111.6610 | 134.8168 | -6.6816 |
| 80 | 1996 | 111.5879 | 134.7581 | 3.3496 |
| 81 | 1997 | 111.5146 | 134.6997 | -2.6706 |
| 82 | 1998 | 111.4411 | 134.6415 | -12.3368 |
| 83 | 1999 | 111.3674 | 134.5835 | -3.0674 |
| 84 | 2000 | 111.2934 | 134.5256 | -0.4598 |
| 85 | 2001 | 111.2193 | 134.4680 | 5.6983 |
| 86 | 2002 | 111.1450 | 134.4106 | 5.1163 |
| 87 | 2003 | 111.0705 | 134.3533 | -2.9463 |
| 88 | 2005 | 110.9209 | 134.2394 |  |
|  |  |  |  |  |

Sum of Residuals
Sum of Squared Residuals
Predicted Residual SS (PRESS) 2787.93204 2921.16963

