

Name: \_\_\_\_\_ ID \_\_\_\_\_

There are 16 questions. Mark answers on the answer sheet at the end of the exam and also circle the answer on the exam. You may submit your scratch paper if you expect to ask for partial credit. Tables are distributed separately.

**1** Here are two probability tables. In each table compute  $P(A|B)$ ,  $P(A)$ , and  $P(B)$  and answer the next two questions.

<b>Table I</b>	$B^c$	B	<b>Table II</b>	$B^c$	B
$A^c$	0.20	0.05	$A^c$	0.06	0.14
A	0.20	0.55	A	0.24	0.56

In which table(s) is A independent of B?

- 1.1** a) Table I    b) Table II    c) Neither    d) Both

In Table I which of the following is true?

- 1.2** a)  $P(A | B) > P(A)$     b)  $P(A | B) < P(A)$     c)  $P(A | B) = P(A)$

**2** A diagnostic test for disease D has sensitivity  $P(+ | D)=0.98$  and specificity  $P(- | \sim D)=0.99$ . The incidence (prevalence) of the disease (D) is  $P(D)=0.01$ . Use Bayes rule to compute  $P(D | +)$  and  $P(\sim D | -)$ .

- 2.1**  $P(D | +)$  is    a)  $\approx 0.5$     b)  $\approx 0.6$     c)  $\approx 0.7$     d)  $\approx 0.8$     e)  $\geq 0.9$

- 2.2**  $P(\sim D | -)$  is    a)  $\leq 0.97$     b) 0.98    c) 0.998    d) 0.9998    e)  $\geq 0.9999$

**3** Make a stemplot (stem-and-leaf diagram) of the following observations (do not split the stems). Compute the five number summary (use the instructor's definition of quartiles).

3.5 5.7 6.3 5.1 5.5 6.2 5.2 4.4 3.5 3.9 7.1 1.5  
5.5 2.8 7.6 6.1 3.3 2.9 4.4 5.0 4.1 6.6 3.6

Stems	Leaves	5 Number summary

4 Compute the mean and standard deviation of these numbers: 63, 48, 55, 62, 52, 44. Show your work on the back.

$\bar{x} =$	
$s =$	

5 Suppose that  $P(A) = .4$  and  $P(B) = .5$ , and  $P(A \cap B) = .20$ .

	<b>A</b>	<b>~ A</b>	
<b>B</b>			
<b>~ B</b>			



5.1 Fill in the joint and marginal probabilities in the table.

5.2 Are A and B independent? No  Yes  Why? \_\_\_\_\_

6 Mom had the frugality of most people who lived through the great depression. For years she used an old dented tin measuring cup. I told my dad that it wouldn't measure properly, but he solemnly assured me that the dent wouldn't change the capacity of the cup as long as the metal was not broken. I believed him for a long time until one day I pictured in my mind what would happen if I totally flattened the cup – it would hold next to nothing.

What is the name of this type of reasoning? What is the purpose?

6.1 Name: \_\_\_\_\_

6.2 Purpose: \_\_\_\_\_

7 Give an operational definition of the concept of the subjective probability of a sentence such as, "A woman will be elected president of the U.S. before 2025."

\_\_\_\_\_

\_\_\_\_\_

8 As of 9/26/04 the asking prices on TradeSports.comj are \$6.91 to place a bet that pays \$10 if Bush wins the election and \$3.30 to place a bet that pays \$10 if Kerry wins. Suppose you make both bets. Why is that irrational? What is the name of the type of financial situation that you would have placed yourself in?

8.1 Why irrational? \_\_\_\_\_

8.2 Name? \_\_\_\_\_

9 Here are Jake's fair prices for various bets. Which two prices are incoherent? What should they be? What law of probability tells you this?

**Jake's Betting Layout**

<i>All bets pay \$1</i>	<b>H<sub>R</sub></b> price: 54¢	<b>H<sub>D</sub></b> price: 46¢
<b>S<sub>R</sub></b> price: 21¢	<b>H<sub>R</sub> ∩ S<sub>R</sub></b> price: 10¢	<b>H<sub>D</sub> ∩ S<sub>R</sub></b> price: 7¢
<b>S<sub>D</sub></b> price: 79¢	<b>H<sub>R</sub> ∩ S<sub>D</sub></b> price: 40¢	<b>H<sub>D</sub> ∩ S<sub>D</sub></b> price: 39¢

<b>H<sub>D</sub> ∩ S<sub>D</sub></b> price: 90¢
--

9.1 First wrong price \_\_\_\_\_ Correct Value \_\_\_\_\_

Why \_\_\_\_\_

9.2 Second wrong price \_\_\_\_\_ Correct Value \_\_\_\_\_

Why \_\_\_\_\_

10 There are three boxes. A, B, and C. Box A contains 5 B and 15 W chips. Box B contains 10 B and 10 W chips. Box C contains 17 B and 3 W chips. Someone selects a box at random and hands it to you. You can't look in the box, but you can draw four chips from the box (without replacement). You made  $n = 4$  draws and got 1 black chip and 3 white chips. Compute the posterior probabilities of the three models (A, B, and C) given the data. Label each column with a word or phrase and also with a probability expression (for example column 2 is the Prior and the probability expression is  $P(\text{Model})$ ). You may use M and D to stand for the words "model" and "data." Fill in the correct word in the probability expression under the table as well.

	<b>Prior</b>			
<b>Model</b>	<b>P(Model)</b>	<b>P(_____)</b>	<b>P(_____)</b>	<b>P(_____)</b>
<b>A (5B 15W)</b>				
<b>B (10B 10W)</b>				
<b>C (17B 3W)</b>				

<b>P(_____)</b>
-----------------

→

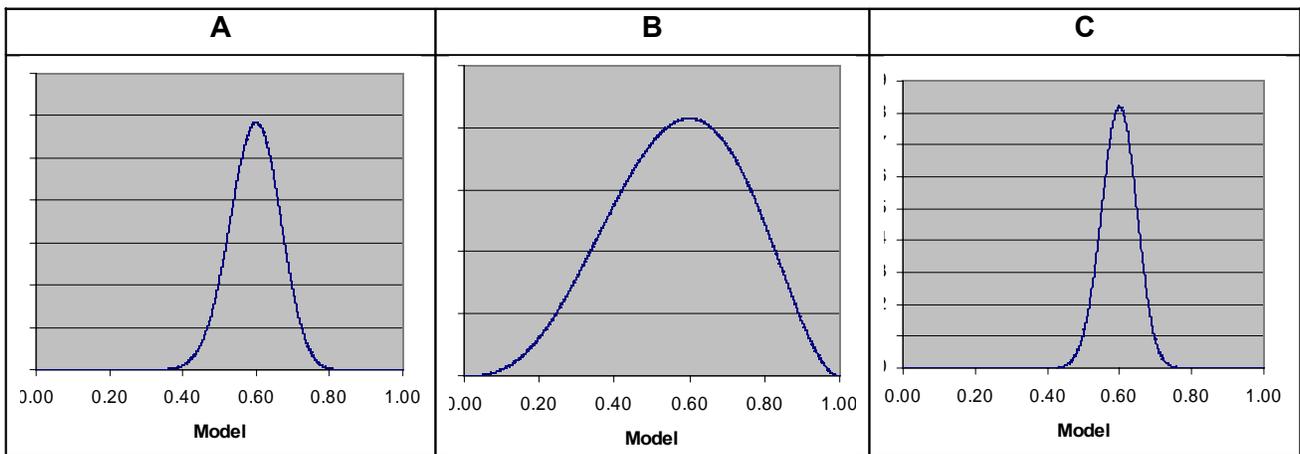
--

**11** MnSOST-R is a diagnostic test to predict if a paroled sex offender will recidivate; i.e., commit another violent sexual offense ( V ) sometime in the future. The sensitivity of the test is  $P(+ | V) = 0.17$  and the specificity is  $P(- | \sim V) = 0.97$  . Assuming that the prevalence of violent sexual recidivism is  $P(V) = 0.30$ , use Bayes rule to compute  $P(V | +)$  and  $P(\sim V | -)$ . Show your work on the back.

**11.1**  $P(V | +) =$  \_\_\_\_\_

**11.2**  $P(\sim V | -) =$  \_\_\_\_\_

**12** Here are three posterior distributions produced by the spreadsheet Bernoulli.xls. Each was obtained by spinning a penny n times and observing the number of heads. One graph is for n = 5 and x=3, one is for n = 20 and x = 12, and one is for n = 100 and x = 60. Say which is which, and match it to one of the credible intervals shown below



Here are the 95% credible intervals (but not in the same order as the pictures).

I	II	III
0.24 to 0.90	0.45 to 0.74	0.50 to 0.69

**12.1** Results for 3 heads in 5 spins are: Graph: \_\_\_\_\_ Credible Interval: \_\_\_\_\_

**12.2** Results for 12 heads in 20 spins are: Graph: \_\_\_\_\_ Credible Interval: \_\_\_\_\_

**12.3** Results for 60 heads in 100 spins are: Graph: \_\_\_\_\_ Credible Interval: \_\_\_\_\_

## Numbered Equations

$$P(E|R) = \frac{P(E \cap R)}{P(R)}$$

$$P(E \cap R) = P(E|R) \cdot P(R)$$

$$P(R \cup E) = P(R) + P(E) - P(R \cap E)$$

$$P(\sim R) = 1 - P(R)$$

The sentences  $R$  and  $E$  are said to be statistically *independent* if knowing that  $R$  is true does not change the probability that  $E$  is true. Thus, the sentences  $R$  and  $E$  are independent if and only if

$$P(E|R) = P(E). \quad (2.7)$$

$$\text{fair price of a standard bet} = \$P(\text{Win})$$

$$\text{expected value} = E(V) = \sum_{i=1}^k V_i \cdot P_i$$

$$\begin{array}{r} P(H_R \cap S_R) + P(H_D \cap S_R) = P(S_R) \\ + \quad + \\ P(H_R \cap S_D) + P(H_D \cap S_D) = P(S_D) \\ \parallel \quad \parallel \\ P(H_R) \quad P(H_D) \end{array}$$

$$P(H_D \cup S_D) = P(H_D) + P(S_D) - P(H_D \cap S_D)$$

$$P(H_D \cup S_D) + P(H_D \cap S_D) = P(H_D) + P(S_D)$$

$$P(\Omega) = 1$$

$$P(\emptyset) = 0$$

$$P(S^c) = 1 - P(S)$$

$$\text{fair price of } \{\$M \text{ if } S\} = \$M \cdot P(S)$$

$$\text{bookmaker's price for } \{\$1 \text{ if } S\} = \frac{\text{gambler's stake}}{\text{gambler's stake} + \text{bookmaker's stake}}$$

$$f = P(R \cap D) + f \cdot [1 - P(D)]$$

$$P(R \cap D) = f \cdot P(D)$$

$$P(R \cap D) = P(R|D) \cdot P(D)$$

$$P(R|D) = \frac{P(R \cap D)}{P(D)}$$

If sentences  $T$  and  $W$  are independent,

$$P(T \cap W) = P(T) \cdot P(W)$$

$$EV = \sum_{i=1}^k V_i \cdot P(S_i)$$

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n} = \frac{\text{Sum}}{n}$$

$$s = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{\text{CSS}}{n-1}} = \sqrt{\frac{\text{centered sum of squares}}{n-1}}$$

$$P(M_j | D) = \frac{P(M_j \cap D)}{P(D)} = \frac{P(D | M_j) \cdot P(M_j)}{\sum_{\text{all models } i} P(D | M_i) \cdot P(M_i)}$$

If  $D =$  "The patient tested positive," the likelihood is  
 $P(D | M_1) = P(+ | \text{Infected}) = \text{sensitivity}$   
 $P(D | M_2) = P(+ | \text{Not infected}) = 1 - \text{specificity}$

If  $D =$  "The patient tested negative," the likelihood is  
 $P(D | M_1) = P(- | \text{Infected}) = 1 - \text{sensitivity}$   
 $P(D | M_2) = P(- | \text{Not infected}) = \text{specificity}$

$$\begin{aligned} \text{likelihood} &= P(x \text{ heads in } n \text{ spins} \mid \text{heads rate} = p) \\ &= p^x \cdot (1-p)^{(n-x)} \end{aligned}$$